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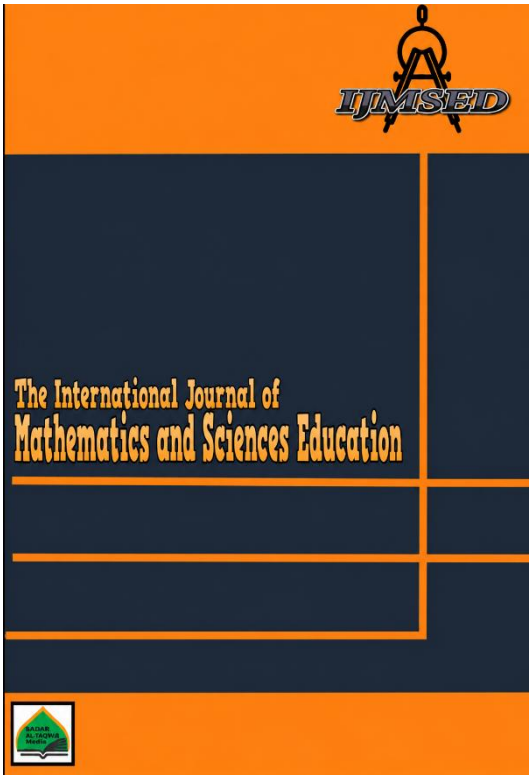

Analysis of junior high school students' probabilistic reasoning in solving probability problems as a basis for the development of E-learning probability (E-PROF)

Imam Rofiki[1], Puguh Darmawan[2*], Siti Faizah[3], Tuti Mutia[4], Liny Mardhiyatirrahmah[5], Sri Wahyuni[6], Mutiara Sani[7], Syekha Vivi Alaiya[8], Feti Okta Lestari[9], Fahrany Wahyu Andini[10], Leyna Dwi Agustina[11]

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Abstract
Studies of probabilistic reasoning describe how students use intuition, counts, and formal calculations, while research on digital probability learning examines modeling and multiple representations. The link between a specific reasoning difficulty and the interaction a learner should perform remains under-specified. This descriptive qualitative study examined how Grade 9 students justified single-draw probabilities and used the findings to derive design requirements for E-Learning Probability (E-PROF). Sixteen students completed a written test, and four focal cases representing the Jones framework categories were analyzed through written responses and interview excerpts. The subjective case used intuitive percentages. The transitional case wrote the correct fraction but gave an ambiguous explanation of its meaning. The informal quantitative case counted an added blue ball but retained the original total, whereas the numerical case used the new total for both probabilities. These responses informed four proposed features: visual prediction and counting, prompts for interpreting probability notation, linked visual and symbolic representations, and enrichment through comparisons across compositions. A specific design requirement is to have learners predict and justify the revised denominator and both fractions before viewing the updated representations. The study provides a basis for prototype design and subsequent classroom evaluation.

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1. Introduction

Probability is an important topic in mathematics because it relates to many situations involving uncertainty and decision-making (Supratman et al., 2025). However, probability remains difficult for many students. The difficulty lies not only in applying formulas but also in students' ability to understand situations, compare the likelihood of events, use quantitative information, and explain the reasoning behind their answers (Batanero & Álvarez-Arroyo, 2024; Begolli et al., 2021). Proportional reasoning is also associated with the development of students' probabilistic knowledge (Begolli et al., 2021). In Indonesian secondary classrooms, cooperative instruction on probability has improved students' achievement and mathematical disposition, although their strategic flexibility developed more slowly than their accuracy (Mayangsari et al., 2025). Therefore, understanding students' ability to solve probability problems requires attention to the reasoning processes they use during problem solving.

Probabilistic reasoning involves the ability to understand uncertainty, compare the likelihood of events, use quantitative information, and provide reasons for a conclusion (Batanero et al., 2023). Students' reasoning does not always develop formally, as intuition, experience, and reasoning biases may influence how they make decisions in probabilistic situations (Batanero & Álvarez-Arroyo, 2024). Rubel (2007) also found that middle and high school students may use different strategies and justifications when solving probabilistic tasks. Four levels of probabilistic reasoning have been identified among students: *subjective, transitional, informal quantitative,* and *numerical* (Fa'ani et al., 2022; Jones et al., 1999; Kingston & Twohill, 2022; Taram et al., 2020). These levels describe the development of reasoning from the use of intuition and experience toward the use of quantitative relationships and more formal reasoning.

The position of the current study is presented in Table 1. The table illustrates the relationship between previous studies and the focus and novelty of the current study.

Table 1

Research Position

Previous Studies	Research Focus	Main Findings
Batanero & Álvarez-Arroyo (2024); Begolli et al. (2021); Rubel (2007)	Probability learning difficulties and probabilistic reasoning	Students experience difficulties in understanding probability and may use different intuitive, quantitative, and formal strategies when solving probability problems.
Fa'ani et al. (2022)	Levels of probabilistic reasoning	Identified four levels: *subjective*, *transitional*, *informal quantitative*, and *numerical*.
Kazak & Pratt (2021); Post & Prediger (2024)	Modeling, simulation, and multiple representations	Modeling, simulation, and multiple representations support students in connecting experiences and mathematical probability concepts.
Present study	Students' probabilistic reasoning in solving probability problems and its implications for E-PROF development.	Derives a predict-and-justify interaction from the observed failure to update the total.

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Existing studies describe levels of probabilistic reasoning (Fa'ani et al., 2022) and examine how modeling and connected representations can support probability instruction (Kazak & Pratt, 2021; Post & Prediger, 2024). The present study links those strands by using specific student responses to specify what learners should do before receiving feedback in a digital environment. In one focal case, S2 counted the additional blue ball but retained the original denominator for both probabilities. This response motivates a design requirement that extends the general use of connected representations to a specific interaction: students predict and justify the revised total and both fractions before the simulation reveals the linked counts and symbols.

E-Learning Probability (E-PROF) is proposed as a digital learning environment that uses visual simulations, prompts, feedback, and reflection to support probability problem solving. The design requirements below specify how these features could respond to differences observed in students' reasoning (Batanero & Álvarez-Arroyo, 2024; Kazak & Pratt, 2021; Weinhandl, Baldinger, & Kapplmüller, 2025; Weinhandl, Baldinger, & Riegler, 2025). Previous studies support this direction: interactive, visually supported software improved students' achievement and retention in learning integers (Dogbe & Ali, 2026), and an offline guided inquiry e-module developed through the ADDIE model was rated valid and practical and was associated with gains in junior high school students' critical thinking (Handayani et al., 2026).

This study examined how Grade 9 students reasoned about the probability of an event and compared probabilities. It then used four focal cases, interpreted through the Jones et al. (1999) framework, to derive design requirements for E-PROF.

2. Method

2.1 Research Design

This study used a descriptive qualitative method to examine, describe, and analyze how junior high school students reason when solving probability problems. The results of this analysis were used to derive implications and design requirements, which then formed the basis of the E-Learning Probability (E-PROF) application.

2.2 Research Subjects

Sixteen Grade 9 students from one class at a junior high school completed a probability problem-solving test. The profile mapping classified two students as subjective, two as transitional, three as informal quantitative, and nine as numerical. Four focal cases were selected purposively, one per category, using the reported criterion of clarity of written responses and initial explanations. The focal cases were S12, S6, S2, and S15, respectively.

2.3 Research Instruments

The researchers interpreted students' written solutions and explanations using the four-level rubric in Table 2. An open-ended probability test elicited students' initial comparisons and justifications. For example, students were asked which color was more likely to be drawn from a box containing three red balls and one blue ball, and why. The in-depth interview prompts probed how students determined probabilities, interpreted

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a fraction such as 3/4, and revised a probability comparison after the box contents changed. The instruments were reviewed by two mathematics education experts.

Table 2

Probabilistic Reasoning Leveling Rubric

Level	Category	Description of Probabilistic Reasoning
4	Numerical	Students correctly use sample space information, calculate probabilities accurately, compare them using quantitative measures, and show consistent mathematical reasoning.
3	Informal Quantitative	Students use quantitative thinking to determine or compare chances, but they do not always apply numerical methods fully or in a formal way.
2	Transitional	Students are starting to use quantitative information, but their reasoning still mixes in subjective or intuitive ideas, so their results and explanations are not yet consistent.
1	Subjective	Students determine probabilities using experience, intuition, perception, or personal beliefs, without fully considering the underlying probability structures.

2.4 Data Collection

The written problem-solving test lasted 40 minutes. The in-depth interview consisted of 20 questions and lasted 30 minutes. The focal-case results draw on written solutions and the interview excerpts in Tables 6–9, which include questions about the probability of an event and probability comparisons.

2.5 Data Analysis

The analysis involved reviewing responses, identifying evidence relevant to the rubric, organizing the evidence by profile, and interpreting the four focal cases. For the reported ball-drawing task, the distinctions depended on how students justified their answers. Intuitive estimates indicated subjective reasoning. A correct fraction paired with unclear interpretation indicated transitional reasoning. Explicit counts paired with an unchanged total after a change in the box indicated informal quantitative reasoning. Recomputing both probabilities from the changed total provided evidence of numerical reasoning. Table 3 connects the rubric categories to preliminary learning needs, and Table 9 links the focal responses to specific design requirements.

Table 3

Implications for E-PROF Development Based on the Characteristics of Probabilistic Reasoning

Level	Category	Characteristics of Probabilistic Reasoning	Students' Needs	Implication of E-PROF Development
4	Numerical	Uses the sample space accurately, calculates probabilities numerically, compares results using quantitative methods, and explains	Provide enrichment, introduce more complex real-world problems, and give students opportunities to apply	E-PROF should provide enrichment questions, real-world problems, challenges at different levels, and reflection activities to encourage deeper thinking.

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Level	Category	Characteristics of Probabilistic Reasoning	Students' Needs	Implication of E-PROF Development
		the reasoning clearly and consistently.	concepts in new situations.	
3	Informal Quantitative	Takes numbers into account when judging or comparing chances but does not yet rely fully on formal calculations or formulas.	Support students in linking visual or concrete examples to numbers and symbols.	E-PROF needs to offer visualizations, simulations, step-by-step examples, and support that help students move from informal to formal reasoning.
2	Transitional	Starts using numbers but still combines them with intuition or personal judgment, so the reasoning is not yet fully consistent.	Strengthen basic ideas about probability and address misunderstandings by allowing students to explore and experiment.	E-PROF should include activities for exploration, trial simulations, direct feedback, guiding questions, and scaffolding for students.
1	Subjective	Determines probability based on experience, intuition, or personal beliefs without fully considering how probability works.	Build a clear understanding of probability through real-life examples and hands-on activities.	E-PROF should start with real-life situations, interactive simulations, visual aids, clear examples, and simple questions before moving on to formal calculations.

2.6 Ethics Approval and Informed Consent

This study received ethical approval from the Institute for Research and Community Service (LPPM), Universitas Negeri Malang, Indonesia. Before data collection, written informed consent was obtained from the parents or legal guardians of the participating Grade 9 students, and assent was obtained from the students. Participation was voluntary, and student codes were used to protect participants' identities when reporting written responses and interview excerpts.

3. Results

3.1 Students' Probabilistic Reasoning Profiles

Table 4 summarizes the reported profile mapping for all 16 students, and Table 5 presents the written test answers of the four focal cases. The written test was administered to the full group, and the interview excerpts reproduced in this article concern the four focal cases.

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Table 4
Result of Students' Profile Mapping

Subject	Level of Reasoning	Reason for Mapping the Subject into the Group
S1	Informal Quantitative	Reported 75% and 25%; the profile summary does not document how these estimates were justified.
S2	Informal Quantitative	Identified the initial red-to-total ratio and increased the blue count after an addition while retaining the denominator of four.
S3	Numerical	Stated the probabilities formally (3/4 and 1/4) and accurately calculated the new probabilities when a ball was added (3/5 and 2/5).
S4	Subjective	Relied on guesses of 90% and 10% and responded "don't know, idk" when asked about the mathematical meaning of probability fractions.
S5	Numerical	Explained probability using fractions (3/4 and 1/4) and derived new probabilities when a ball was added (3/5 and 2/5).
S6	Transitional	Began to recognize the fraction 3/4 but interpreted it intuitively as "there is only 1 possibility that the ball can be drawn".
S7	Numerical	Related the fraction 3/4 to 75% and adjusted the probabilities to 60% and 40% when the number of balls increased.
S8	Transitional	Reported an 80-to-20 comparison; the profile summary does not document the justification.
S9	Numerical	Presented structured computational steps, such as $\frac{3\times25}{4\times25} = 75\%$, to support the probability argument.
S10	Numerical	Wrote accurate fraction-to-percentage conversions, for example, $\frac{3}{4} = 75\%$ and, when the sample changed, $\frac{3}{5} = 60\%$.
S11	Numerical	Described the meaning of the fraction 3/4 based on the total sample and constructed new probabilities (60% and 40%) after a ball was added.
S12	Subjective	Admitted answering "by instinct", gave subjective percentages of 99% and 1%, and stated that he did not understand the influence of the number of outcomes.
S13	Informal Quantitative	Applied ratios (3:1 and 3:2) and basic fraction operations, but the calculations were not yet fully formal.
S14	Numerical	Calculated percentages using sound quantitative methods, such as $\frac{100}{5} = 20\%$, then multiplied $2 \times 20\% = 40\%$ for the blue balls.
S15	Numerical	Used five as the total in both recalculated probabilities and converted the fractions to 60% and 40%, e.g., $\frac{3}{5} \times 100\% = 60\%$
S16	Numerical	Stated the overall probability as 100% and then distributed it as 60% and 40% when the total number of balls increased.

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Table 5
Written Test Answers of the Focal Cases

Subject	Level of Reasoning	Student's Answer in the Written Test	Analysis of the Answer
S12	*Subjective*	The red ball, because there are many red balls.	This choice is based on noticing that there are many red balls, without using exact numbers or comparisons.
S6	*Transitional*	The red ball has a higher probability of being drawn because there are more red balls; the probability P(certainty of being drawn) is 3/4.	This response starts to use the fraction 3/4, but still mixes in everyday or personal reasoning.
S2	*Informal Quantitative*	There is a higher probability of drawing a red ball because there are more red balls than blue balls. There are 3 red balls, making them more likely to be drawn than the 1 blue ball.	The written explanation compares the counts of red and blue balls.
S15	*Numerical*	The red ball has a high probability of being drawn because there are more red balls and a higher percentage (75%), while the blue ball has only 25%.	The reported percentages are correct for the initial composition.

The following analysis uses S12 (subjective), S6 (transitional), S2 (informal quantitative), and S15 (numerical) to illustrate the four reported profiles and derive corresponding requirements for E-PROF.

3.2 Characteristics of Students' Probabilistic Reasoning

The following cases are interpreted using the written responses and in-depth interview excerpts for each student.

Subject with Subjective Level of Probabilistic Reasoning

In the written comparison task, S12 chose red "because there were a lot of red balls" (Figure 1). This response identified the more numerous color and gave a qualitative explanation.

Figure 1
Written Work of S12



Table 5 shows how S12 estimated probabilities and responded when asked to explain the estimate.

Table 5
Interview Excerpt of Subject on Subjective Level

Question (Construct)	Question of Interview	Students' Answer	Analysis of Students' Characteristic
7 (*Probability of an Event)*	What do you think is the probability of a red ball being drawn?	99%	The student gave a very high estimate based on personal judgment (99%), which does not match the actual

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Question (Construct)	Question of Interview	Students' Answer	Analysis of Students' Characteristic
			proportion in the sample space (3 out of 4).
8 (*Probability of an Event)*	What do you think is the probability of a blue ball being drawn?	1%	S12 gave 1%; the accompanying justification is reported in Question 10.
10 (*Probability of an Event)*	How do you determine the probability of drawing a blue ball?	I played it by instinct because there is only one blue ball.	The student clearly admitted that they had relied on "instinct" (that is, subjective intuition) when making their decision about the probability problem.
13 (*Probability of an Event)*	If the probability of a red ball is (3/4), how do you explain the meaning of (3/4)?	I don't know	This shows a failure to understand the meaning of fraction symbols when they are used to represent mathematical probability.
20 (*Probability Comparisons*)	Based on your answer, how does the number of an outcome affect the probability of that outcome occurring?	I don't understand, sir. Sorry.	At this stage, the student does not understand the relationship between the number of outcomes and the magnitude of the resulting probability.

S12 gave estimates of 99% and 1% and explained the blue-ball judgment as "I played it by instinct because there is only one blue ball" (Table 5, Question 10). S12 noticed the single blue ball, but the explanation did not relate that count to the four equally likely draws. S12 also replied "I don't know" when asked to interpret 3/4. This combination of an explicit appeal to instinct and the absence of a part-to-whole justification supports the subjective classification.

Subject with Transitional Level of Probabilistic Reasoning

S6 identified red as more likely and wrote "P (certainly drawn) = 3/4" (Figure 2, translated wording).

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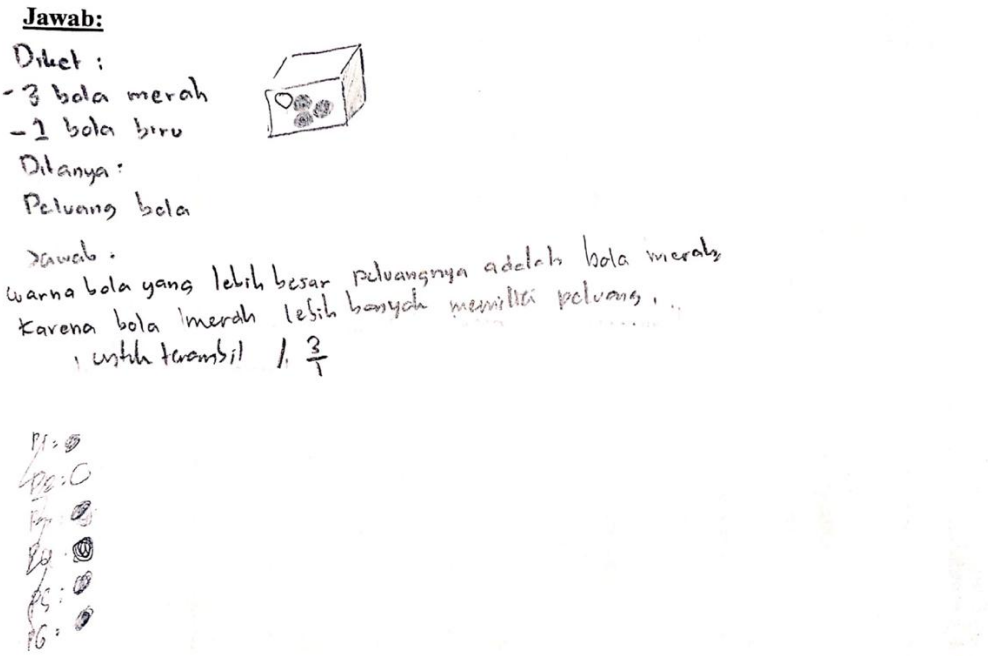
Figure 2

Written Work of S6



The fraction is correct for drawing red, while "certainly drawn" makes the event label ambiguous. The interview probed what S6 understood by 3/4 (Table 6).

Table 6

Interview Excerpt of Subject on Transitional Level

Question (Construct)	Question of Interview	Students' Answer	Analysis of Students' Characteristic
7 (*Probability of an Event)*	What do you think is the probability of a red ball being drawn?	P (certain to be drawn) = 3/4	S6 gives an explanation focused on one possible result or one draw; the response does not identify the meaning of 3 and 4.
13 (*Probability of an Event)*	If the probability of a red ball is (3/4), how do you explain the meaning of (3/4)?	A value of 3/4 indicates that there is just one possible result as to which ball is selected.	This indicates cognitive conflict, a transitional phase. The student recognizes the fraction 3/4 but fails to define the relationship between the numerator (expected outcome) and denominator (sample space). Instead, they interpret the situation intuitively as involving the drawing of a single ball.
20 (*Probability Comparisons*)	How can you tell that a red ball is more likely to be drawn?	Because the red ball has a higher probability.	The explanation remains qualitative and redundant. Instead of referring to the proportional comparison of the number of balls, the student simply repeats that red has a "greater chance."

S6 explained that "there is just one possible result as to which ball is selected" (Table 6, Question 13). Read alongside the correct fraction and the statement that red is more likely, this could refer to drawing one ball or reflect confusion about certainty and probability notation. The excerpt cannot distinguish these interpretations. The

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transitional classification rests on the combination of correct symbolic form and an explanation that does not identify the three favorable outcomes and four possible outcomes.

Subject with Informal Quantitative Level of Probabilistic Reasoning

S2 justified the choice of red by identifying three red balls and one blue ball (Figure 3). This answer uses the observed counts to compare the two colors.

Figure 3

Written Work of S2

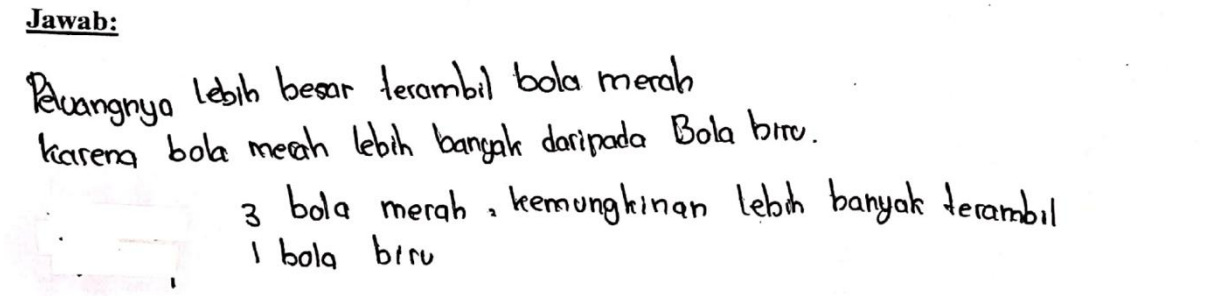

Table 7 shows how S2 used the counts before and after the composition changed.

Table 7

Interview Excerpt of Subject on Informal Quantitative Level

Question (Construct)	**Question of Interview**	**Students' Answer**	**Analysis of Students' Characteristic**
9 (*Probability of an Event)*	How do you determine the probability of drawing a red ball?	If you look at the ratio of the balls there are 3 red balls and the total number of balls is 4, so the ratio is 3:4.	S2 identifies the part and whole in the initial composition.
17 (*Probability Comparisons*)	How significant do you think the difference in probability is between drawing a red ball and a blue ball?	The difference is very significant since the ratio of red balls to blue balls is 3 to 1.	The student logically compares two events in terms of quantitative values, such as a ratio of 3 to 1, which represents a move away from simple subjective guessing toward numerical analysis.
19 (*Probability Comparisons*)	If one blue ball is added, how does the ratio of probabilities for red and blue balls change? Explain.	The chance of drawing a red ball is still 3/4, but that of drawing a blue ball is now 2/4 since one blue ball has been added.	The key difficulty is that the student focuses only on increasing the number of favorable outcomes (adding 1 to the blue balls so that the numerator becomes 2) without updating the denominator (the total sample space, which should be 5).

S2 described the original red-to-total ratio as 3:4. This demonstrates part-to-whole counting in the given box. When one blue ball was added, S2 gave 3/4 for red and 2/4 for

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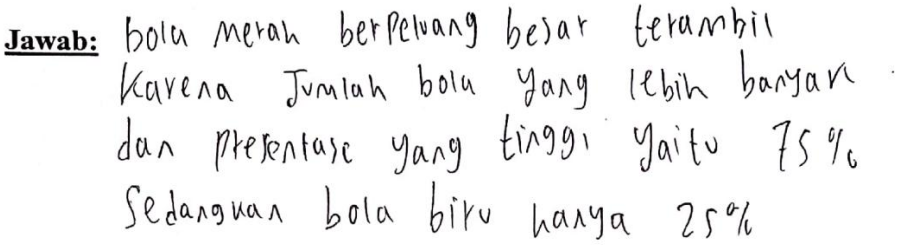
blue. The new blue numerator shows that S2 registered the added ball, while the unchanged denominator shows that the same change was not carried through to the total. This pattern supports the informal quantitative classification for the reported task and leaves flexible proportional reasoning unestablished.

Subject with Numerical Level of Probabilistic Reasoning

In the written task, S15 identified red as more likely and reported 75% for red and 25% for blue (Figure 4).

Figure 4

Written Work of S15



S15 also showed consistent calculation skills during the interview. The transcript of this interview is shown in Table 8.

Table 8

Interview Excerpt of Subject on Numerical Level

Question (Construct)	Question of Interview	Students' Answer	Analysis of Students' Characteristic
9 (*Probability of an Event*)	How do you determine the probability of drawing a red ball?	$3 \times 25\% = 75\%$	The student shows a mature understanding of numerical computation by working out that the probability weight of one ball is 25% and then multiplying this by the number of occurrences (3 red balls), using formal mathematical notation.
13 (*Probability of an Event*)	If the probability of a red ball is (3/4), how do you explain the meaning of (3/4)?	3 = red balls, 4 = total number of balls	The student gives a conceptually accurate definition of the meaning of probability fractions, specifying the numerator as the expected outcome (that is, the object) and the denominator as the total sample space.
16 (*Probability Comparisons*)	Are the probability of getting a red ball and a blue ball the same? Why?	No, because the numbers of balls are different	The student understands that the two probabilities are not equal because they depend on the number of balls of each color in the sample space.
19 (*Probability Comparisons*)	If one blue ball is added, how does the ratio of probabilities for red and blue balls change? Explain.	$\frac{3}{5} \times 100\% = 60\%$ of red balls, $\frac{2}{5} \times 100\% = 40\%$ of blue balls	S15 uses the new total of five for both colors and updates the blue count to two.

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S15 explained the initial 3/4 by identifying 3 as the number of red balls and 4 as the total, and obtained 75% as three shares of 25%. After one blue ball was added, S15 used 3/5 and 2/5 and converted them to 60% and 40%, respectively (Table 8, Question 19). The common denominator of five in both calculations, together with the unchanged red count and increased blue count, shows how S15 incorporated the new total. Question 19 documents this coordination through the calculations.

3.3 Implications for E-PROF Design Requirements

Table 9 links each focal response to a proposed feature and to a learner action that could make the reasoning visible. The proposed features address responses to particular tasks and can be selected according to the explanation a learner gives.

Table 9

Mapping the Implications of Reasoning Characteristics on E-PROF Feature Design

Reasoning profile	Observed response	Learning goal	Proposed feature and learner action
Subjective	S12 gave 99% and 1%, explaining the blue-ball estimate by instinct and the presence of one blue ball.	Connect a qualitative judgment to a part and a whole.	Prompt a prediction, then have the learner count each color and the total and revise the estimate.
Transitional	S6 wrote 3/4 alongside "certainly drawn" and gave an ambiguous explanation of the fraction.	Distinguish the event, the number of draws, and the possible outcomes.	Ask the learner to label 3 and 4, identify the event P(red), and explain whether blue is still possible before feedback.
Informal Quantitative	S2 changed the blue count to two while retaining four as the total in both fractions.	Coordinate the two event counts with their common total.	Add a blue ball and ask the learner to predict and justify the new denominator and both fractions before the linked displays update.
Numerical	S15 recalculated both probabilities using five as the denominator and reported 60% and 40%.	Apply the same reasoning across different compositions.	Present several changes to the box and ask the learner to explain which preserve the probability, then verify with counts and fractions.

4. Discussion

The four focal cases show why identifying the more likely outcome provides limited evidence of probabilistic reasoning. All four students chose red when the box contained three red balls and one blue ball, yet their explanations differed in how they connected favourable outcomes to the total. The framework of Jones et al. (1999) helps explain these differences by directing attention to the reasoning behind an answer. Rubel (2007) likewise examined students' reasoning through their solutions to specific probability tasks. Here, the four profiles describe the reasoning evident in the recorded responses. The study observed students on a set of tasks at one point in time.

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S12 used the greater number of red balls to identify the more likely colour. This comparison shows that intuition can support a sound initial judgment about relative likelihood, a role consistent with the discussion of intuitive probabilistic thinking by Batanero and Álvarez-Arroyo (2024). The difficulty appeared when S12 assigned 99% and 1% without explaining how either count related to the four possible ball outcomes. S6 wrote 3/4, the correct probability of drawing red, but used wording that made the event ambiguous. The interview leaves open whether S6 meant that each draw yields one outcome or that red was certain. Asking students to name the event and explain what each number counts would help distinguish these interpretations.

The change in box composition exposed a different difficulty. S2 used the original ratio of three red balls to four balls in total. After a blue ball was added, S2 increased the blue numerator to two but kept four as the denominator for both colours. S15 used the new total of five and calculated 3/5 and 2/5, equivalent to 60% and 40%. The comparison shows that learners can recognise a part-to-whole relationship in a fixed composition while having difficulty updating the whole when one part changes. Begolli et al. (2021) link proportional reasoning to probability learning and examine self-explanation as instructional support. In this task, explaining why the probability of red decreases even though its count stays at three would reveal whether a learner understands the role of the denominator.

The proposed E-PROF activities can use each response to determine what feedback a learner needs. A learner who relies on intuition could first predict the more likely colour, count the balls by colour, and use those counts to revise the estimate. A learner who supplies 3/4 with an unclear event label could identify the event, link each term of the fraction to the picture, and explain why blue remains possible. These prompts draw on the role of modelling in probability instruction discussed by Kazak and Pratt (2021). They also apply the principle of explicitly connecting representations discussed by Post and Prediger (2024) to a simpler probability task. Comparable benefits of linking a context with its mathematical representation have been reported for an interactive e-module in which students improved most in representing contextual problems as feasible-region graphs (Mansyuriadi et al., 2026), and for technology-assisted project-based learning that strengthened students' mathematical representation in data presentation (Agusta & Setyaningsih, 2026).

For the changed composition, the simulation would show one blue ball entering the box and pause before revealing the revised fractions. Learners would enter the two colour counts, predict the new total, and justify both fractions. If a learner retained four as the denominator, feedback could ask them to recount the entire box and check whether their two fractions sum to one. The linked display would then show how each colour count and the shared total correspond to the numerator and denominator. This sequence gives learners a chance to explain their choice before inspecting the worked representation. It also makes it possible to distinguish a counting error from a failure to update the total.

The order of these steps matters for interpreting the learner's response. Showing the revised fractions immediately could allow a learner to reproduce them without

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explaining the change. Recording the prediction first gives the teacher evidence of which quantities the learner considered and how the explanation changes after feedback. This requirement is consistent with Sutrisno et al. (2026), who found that pre-service teachers' data-scale misconceptions persisted through online peer discussion and recommended that justification be explicitly required rather than left to emerge from interaction.

Learners who update both fractions could compare boxes containing three red and two blue balls, and six red and four blue balls. Explaining why both give the same probability of drawing red would extend the task to equivalent ratios. These design requirements arise from four cases in one Grade 9 class. A prototype evaluation could compare students' initial explanations with their explanations after feedback and use a new task to assess whether they can update probabilities when the composition changes.

5. Conclusion

The four focal cases from a Grade 9 class showed that students could identify the more likely outcome while giving different explanations for their answers. S12 relied on intuition to estimate probabilities. S6 wrote the correct fraction but used an ambiguous event label. S2 recognised the original part-to-whole ratio and updated the count of blue balls after one was added, yet kept the original total as the denominator. S15 used the revised total to calculate both probabilities. Together, these cases show the value of asking students to explain what the numerator and denominator represent and to revisit their reasoning when the composition changes.

The study translates these observations into design requirements for E-Learning Probability (E-PROF). Its main contribution is a proposed simulation sequence in which students predict the new total, calculate and justify both probabilities, and then inspect a display that links the ball counts with the corresponding fractions. This sequence would allow teachers to see whether students update the total when the number of favourable outcomes changes or stays the same. The design was derived from four cases in one class. A classroom evaluation of a working prototype could examine whether the prompts and feedback improve students' explanations and help them apply the same reasoning to new probability tasks.

Limitations

The study involved 16 students in one class and four focal cases. The evidence concerns probability of an event and simple comparisons in a ball-drawing context. The design requirements are conceptual. Classroom evaluation is needed to assess usability, representational understanding, and transfer to new probability tasks.

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Competing Interests
The authors declare that they have no financial or non-financial competing interests relevant to this study.

Ethics Approval
Report the ethics committee's name and approval reference, or the documented exemption. Add details of school permission where applicable.

Informed Consent
For participants who were minors, specify the parent or guardian consent and student assent procedures, as applicable.

Data Availability
Selected anonymized written responses and interview excerpts supporting the findings are presented in the article.

Author Contributions
Imam Rofiki: Conceptualization and supervision. Formulated the research focus and supervised the study.
Puguh Darmawan: Methodology, project administration, and writing (original draft). Coordinated the study and drafted the manuscript.
Siti Faizah: Methodology and formal analysis. Helped develop the probability tasks and interview guide and interpret the students' reasoning.
Tuti Mutia: Methodology and writing (review and editing). Helped refine the instruments and reviewed the alignment between the research questions, methods, and findings.
Liny Mardhiyatirrahmah: Conceptualization and visualization. Translated the findings into proposed E-PROF features and specified how ball counts and probability fractions would be displayed together.
Sri Wahyuni: Investigation. Coordinated classroom data collection anadministered the written probability tasks.
Mutiara Sani: Investigation and data curation.
Syekha Vivi Alaiya: Formal analysis.
Feti Okta Lestari: Data curation.
Fahrany Wahyu Andini: Visualization and writing (review and editing).
Leyna Dwi Agustina: Writing (review and editing).

Declaration of Generative AI Use
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