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Original Article

Didactic Design of Mathematics Learning to Improve Conceptual Understanding of Triangles and Quadrilaterals Based on Green Numeracy at a Private Islamic Junior High School

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Abstract

Students still experience learning obstacles in understanding the concepts of triangles and quadrilaterals, particularly in distinguishing base and height, applying area formulas to non-routine problems, and connecting learning with real-life contexts, resulting in a gap between procedural and conceptual knowledge. This study aims to describe the learning obstacles experienced by students in understanding the concepts of triangles and quadrilaterals, and to design and test the effectiveness of a green numeracy-based didactic design in improving students' conceptual understanding. This study employed a descriptive qualitative approach with a Didactical Design Research (DDR) design consisting of prospective, metapedadidactic, and retrospective analysis stages. Participants were 33 seventh-grade and 32 eighth-grade students at a private Islamic junior high school in East Lombok Regency, aged 12–14 years, selected through purposive sampling. Instruments included a semi-structured interview guide, an observation sheet, a five-item written test, and a document checklist. Data were analyzed using a qualitative descriptive approach with data reduction (both in sample and in material), data display, linkage with the Theory of Didactical Situations, triangulation and conclusion. The results showed that students experienced ontogenic, epistemological, and didactical obstacles, reflected in a gap between memorized formulas and conceptual understanding; following the implementation of the green numeracy-based didactic design, significant improvements occurred in conceptual understanding, learning engagement, and student confidence; and learning that connected real-world measurement with the surrounding environment was shown to encourage students to construct understanding independently rather than merely repeating formulas. This study concludes that the green numeracy-based didactic design effectively addresses learning obstacles in triangles and quadrilaterals. Future researchers are recommended to expand the participant pool, extend the implementation duration, and integrate technology as well as more diverse environmental contexts.

Keywords: didactical design research; green numeracy; learning obstacle; triangle and quadrilateral

1. Introduction

Conceptual understanding of triangles and quadrilaterals remains a genuine challenge across various levels of education. This is in accordance with the results of research from Lutfi, Juandi and Jupri (2021) where students are faced with an ontogenic obstacle that is a framework structure related to understanding knowledge about relations between elements in plane figures. This conclusion was supported by Bintara and Prabawanto (2024) that stated ninth-grade students have a didactical obstacle in triangular concepts for non-routine problems due to incomplete-able presentation of the material; Soneta and Prabawanto (2024) said as well but in quadrilaterals area concept. Durmaz and Işıksal Bostan (2022) reported that shortcomings in the understanding of the triangle area concept still persisted at the pre-service teacher level, as did Guo (2022) demonstrated with respect to pre-service mathematics teachers who found it difficult to solve word problems concerning triangles, an impediment which has the potential to perpetuate through different educational levels by poor teaching practices. In addition to studies that focus on quadrilaterals themselves, two other studies have also mentioned related findings: one is the study by Ayan-Civak, Baktemur, and Işıksal-Bostan (2021), and the other is the study by Ekawati et al. (2023). Both studies point out that a large number of studies throughout history have recorded the same universal phenomenon.

Moreover, many prior studies have already identified multiple causes for this type of teaching difficulty. First, didactical factors in the form of conventional teaching approaches with minimal contextual variation, as found by Genc and Ergan (2022), who state that geometry instruction without deliberately designed didactical situations tends to fail to trigger the cognitive conflict necessary for building understanding. Naufal, Abdullah, Osman, Abu, Ihsan, and Rondiyah (2021) further assert that without teaching strategies that account for students' geometric thinking stages, such as in the Van Hiele model, students tend to remain at the visual level of thinking without reaching the analysis or informal deduction levels. Second, epistcientific obstacles in the form of weak understanding of geometric conceptions themselves which as stated by Yenil, Arslan, and Broutin (2023) are due to students being trained to memorize formulas without grasping why they work and how the logical structure works behind the formulas and this was aligned with Hendriyanto et al. Third, one of the contextual factors announcing a loose connection between mathematical content and students real-life experiences (Delamontano et al. 2023), that further grows in importance when we consider that conventional mathematics curricula are not providing opportunities for learners to link abstract concepts to aspects of their environment, as also discussed in the systematic review by Vásquez et al. (2023) revealing still limited work around integration of education for sustainable development into school mathematics.

Based on the identification of these problems and their contributing factors, this study offers a solution in the form of a didactic design for mathematics learning on the topic of triangles and quadrilaterals, developed through the Didactical Design Research (DDR) approach and integrating a green numeracy context, namely a numeracy approach that directly connects mathematical concepts with environmental issues and activities in students' lives, such as land measurement, roof-shape observation, and floor-area calculation.

This solution differs fundamentally from previous approaches. They are trained on data until 2023–10 (Genc and Ergan, 2022; Yenil et al. Researches like (2023) was conducted on didactic designs using Theory of Didactic Situations in triangle inequality, but it has not been bound to a real life context but rather only against pure mathematical situations. Akar and Işıksal-Bostan (2024) investigated the didactic transposition of quadrilaterals in the curriculum, although their focus was on a textbook analysis not that of a direct instructional intervention. Shahidayanti, Prahmana dan Fran (2024) created an Ethno-Realistic Mathematics Education-based geometry learning module that was oriented only towards linking the local culture of each human's life to local cultural aspects without touching the environmental problems surrounding it. However, in contrary to this present study, Nursyahidah et al. (2023), the lesson study and STEm quadrilateral learning is based on—was a project that was not an environment-based problem, but engineering. Simultaneously, no research has aimed at a digital collaboration-didactical design which examined new ideas for characterising active learning approaches (p. 2, Lindenbauer, Infanger, & Lavicza, 2024) or that studied physical motion and games as effective attributes to enhance the long-lasting knowledge of students on sustainability problems without attaching it specifically in environmental-related issues (Vale & Barbosa, 2023). Regarding research on green numeracy, Delamontano et al. (2025) did treat the interconnection of literacy and environmental issues but this was for basic number-operation material for elementary school students–not plane geometry topics, and it did not use a DDR framework. Thus, this study offers a novelty in the form of integrating the DDR framework with a green numeracy approach specifically on the topic of triangles and quadrilaterals, which has not been found in previous studies.

Based on the above discussion, this study aims to describe the learning obstacles experienced by students in understanding the concepts of triangles and quadrilaterals, and to design and test the effectiveness of a green numeracy-based didactic design in improving students' conceptual understanding. Based on this aim, the study seeks to answer the following research questions:

1.1. What learning obstacles do students experience in understanding the concepts of triangles and quadrilaterals?
1.2. How effective is a green numeracy-based mathematics didactic design in improving students' conceptual understanding of triangles and quadrilaterals?

2. Method

2.1 Research Design

This study used a descriptive qualitative approach with a Didactical Design Research (DDR) design. This design was chosen because the aim of the study was not only to describe students' learning obstacles but also to design, implement, and evaluate an instructional design cyclically, a characteristic that neither ordinary descriptive designs nor pure experimental designs can accommodate. The phenomenological-hermeneutic approach used by Isnawan, Suryadi, and Turmudi (2022) to uncover how students construct meaning of a mathematical concept served as an important reference in understanding how learning obstacles need to be explored in depth before a design is developed. DDR comprises three interrelated stages: prospective analysis, which identifies learning obstacles before the design is developed; metapedadidactic analysis, which observes students' interaction with the designed didactical situation; and retrospective analysis, which evaluates the correspondence between the design and the implementation outcomes. The choice of DDR was also supported by the findings of Hortelano and Prudente (2024), who showed that approaches based on the Theory of Didactical Situations are effectively used across educational levels to uncover the sources of students' difficulties while simultaneously designing responsive instructional solutions, as further emphasized by Suryadi, Herman, and Prabawanto (2022), who argued that didactical obstacles in mathematics learning need to be systematically analyzed as a basis for developing new instructional designs.

2.2 Participants

The participants of this study were all seventh- and eighth-grade students at a private Islamic junior high school (madrasah tsanawiyah) in East Lombok Regency, comprising 65 students in total (33 seventh-grade students and 32 eighth-grade students), aged approximately 12 to 14 years, consisting of both male and female students in regular classes without any ability-based grouping. The eighth-grade class was selected as the obstacle-testing class because these students had already received instruction on triangles and quadrilaterals in the previous semester, allowing their learning obstacles to be identified retrospectively from their prior learning experience, while the seventh-grade class was selected as the implementation class because these students had not yet received instruction on triangles and quadrilaterals, allowing the effectiveness of the didactic design to be tested from the very beginning of instruction. The school was selected purposively because the researcher was already familiar with the field conditions through prior teaching practicum (PPL) and had previously identified students' learning obstacles in this topic. Such a purposeful selection of participants is consistent with Ahmad and Wilkins's (2024) proposed purposive sampling framework, whereby this strategy is best suited when information-rich participants are required whose experiences or characteristics will inform the research question, thus providing richer and more detailed data than random sampling.

2.3 Research Procedures

The procedure follows three iterative stages of Didactical Design Research: prospective analysis (identifying students' learning obstacles), metapedadidactic analysis (implementing the design and observing classroom interaction), and retrospective analysis (evaluating the design against the predicted outcomes). The research procedure followed the flow shown in the Figure 1, beginning with the prospective analysis stage to identify learning obstacles (ontogenic, epistemological, and didactical) in the topic of triangles and quadrilaterals, in line with the obstacle categorization proposed by Lutfi, Juandi, and Jupri (2021) and Prabowo, Suryadi, Dasari, Juandi, and Junaedi (2022), who emphasized the importance of identifying obstacles as a basis for instructional design. The results of this identification then served as the basis for developing a didactic design in the form of a green numeracy-based Student Worksheet (LKPD), constructed with attention to the predicted student responses to the designed didactical situation, following the approach used by Hariyani, Herman, Suryadi, and Prabawanto (2022) in redesigning instruction based on identified learning obstacles. In the subsequent stage, metapedadidactic analysis, the design was implemented in a lower grade over three sessions, a duration aligned with the school's allocated instructional time for the triangle and quadrilateral unit, to directly observe students' interaction with the designed learning situation. The final stage, retrospective analysis, was conducted by comparing the implementation results against the hypothetical predictions formulated beforehand, in order to assess the design's effectiveness in overcoming students' learning obstacles, as recommended within the DDR methodological framework by Sidik, Suryadi, and Turmudi (2021), who applied a similar retrospective stage in evaluating an obstacle-based instructional design.

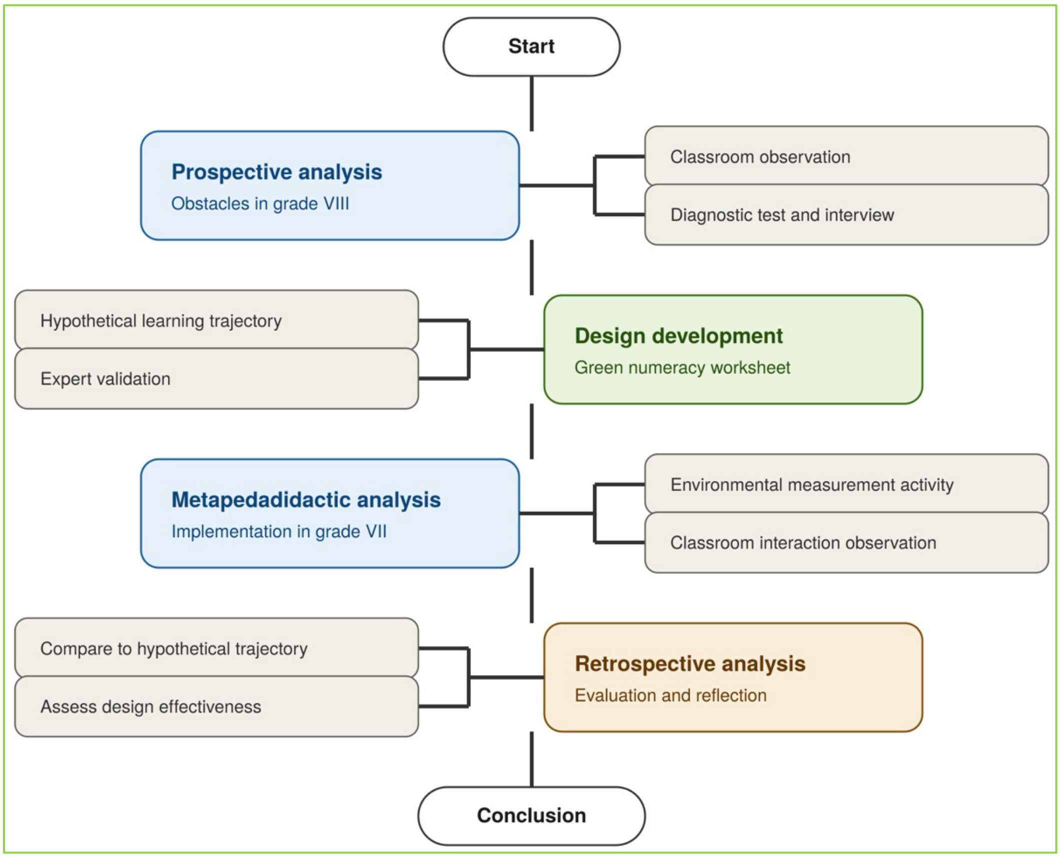


Figure 1 *Research Procedure*

2.4 Data Sources and Data Collection Techniques

This study used a semi-structured interview guide to explore students' understanding of the concepts of triangles and quadrilaterals in depth, along with the reasoning behind their answers, as semi-structured interviews allow the researcher to confirm students' thought processes that are not visible from written work alone, as emphasized by Ahmad and Wilkins (2024), who note that this technique provides both flexibility and depth of data that closed instruments cannot capture, and in line with the interview strategy developed by Marfuah, Suryadi, and Turmudi (2023) for diagnosing students' learning obstacles with greater precision. In addition, an observation sheet was used to record students' behavior, interaction, and responses directly during the learning process, so that the data obtained were contextual and naturalistic, along with a five-item mathematics test on triangles and quadrilaterals used to identify the types and patterns of students' conceptual errors in a measurable way, following the diagnostic testing approach used by Lutfi, Juandi, and Jupri (2021) for the same topic. Another instrument used was a document checklist, which served to verify the completeness of supporting data such as students' work and documentation of learning activities, as similarly used by Marfuah, Suryadi, Turmudi, and Isnawan (2022) in verifying data on teachers' knowledge transposition during online learning. This combination of four instruments was chosen so that the data obtained would complement one another and enable methodological triangulation to strengthen the validity of the research data.

2.5 Data Analysis Techniques

The data analysis technique used was qualitative descriptive analysis with a didactic analysis approach, chosen because it corresponds to the characteristics of the DDR design, which requires analysis at each research stage to be conducted separately yet remain interconnected. The analysis process began with reducing the data obtained from observation, interviews, and tests into categories of learning obstacles (ontogenic, epistemological, and didactical) followed by displaying the reduced data in descriptive narrative form to facilitate the interpretation of findings. These sets of data were then associated with the Theory of Didactical Situations to create meaningful proposed alternative instructional designs which corresponds to the findings by Hortelano and Prudente (2024) in their metasynthesis study on the implementation of this theory at all educational levels. Data validity was examined through triangulation, namely comparing results from various data sources and different data collection methods to ensure consistency of findings, before the researcher ultimately formulated conclusions regarding the effectiveness of the didactic design based on all data that had been analyzed and validated.

2.6 Research Ethics

This study observed a number of qualitative research ethics principles, beginning with obtaining official permission from the school prior to data collection, as well as explaining the purpose and procedure of the research to teachers and students through an informed consent process, including students' right not to participate without any consequences. This principle aligns with the view of Taquette and Souza (2022) that participant consent in qualitative research is an ongoing process that must be maintained throughout the research, not merely obtained at the outset. Students' identities were kept confidential through the use of pseudonyms or initials in reporting the research results, while students' involvement in this study was voluntary and did not disrupt regular classroom instruction, as emphasized by Taquette and Souza (2022), who note that researchers must remain sensitive to participants' potential discomfort and must consistently ensure that the research process does not harm those involved. The data obtained from this study were used solely for academic purposes.

3. Results and Discussion

3.1 Results

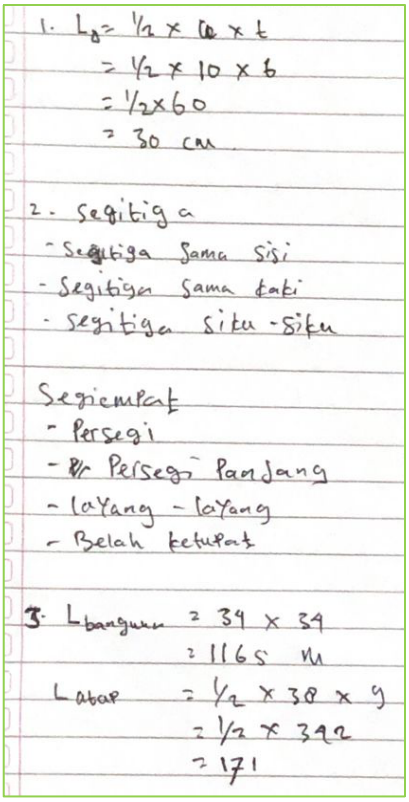
3.1.1 Condition of Mathematics Instruction at the Madrasah

Initial observations conducted by the researcher prior to the main data collection stage revealed that the mathematics instruction at the research site remained dominated by conventional methods, namely one-way lecturing and question-and-answer sessions, with the teacher as the center of learning activities. The instructional media used were also limited to the whiteboard, textbooks, and worksheets, with little involvement of surrounding environmental context or concrete instructional media capable of visualizing abstract concepts. This condition contributed to low student participation; students tended to be passive, quickly lost interest in learning, and more often copied formulas without attempting to understand the underlying meaning and logical relationships.

Specifically regarding the topic of triangles and quadrilaterals, the majority of students still struggled to distinguish between base and height, frequently made errors in applying area and perimeter formulas (particularly when plane figures were presented in unusual or rotated orientations) and had not yet fully and comprehensively understood the properties of these plane figures. Although the teacher had attempted to assist students through additional practice problems and repeated exercises, the conventional approach used, which had not yet connected the concept to everyday life, meant that the learning still felt abstract and lacked meaning for most students. Specifically regarding the topic of triangles and quadrilaterals, most of the 32 eighth-grade students still struggled to distinguish between base and height, frequently made errors in applying area and perimeter formulas (particularly when plane figures were presented in unusual or rotated orientations), and had not yet fully and comprehensively understood the properties of these plane figures.

3.1.2 Description of Students' Learning Obstacles

Based on data collected through classroom observation, analysis of student worksheets, and in-depth interviews, the researcher identified various forms of learning obstacles experienced by students in understanding the concepts of triangles and quadrilaterals. These obstacles consistently emerged across three main processes: when students solved computational problems, when they attempted to grasp the meaning of a geometric concept, and when they tried to connect visual representations (images of plane figures) with the corresponding mathematical formulas. As a concrete illustration, one student with the initials EM appeared to struggle with determining the area of a triangle and the process of calculating it, as shown in the following student worksheet (see Figure 2).


1. $L_\Delta = \frac{1}{2} \times a \times t$
$= \frac{1}{2} \times 10 \times 6$
$= \frac{1}{2} \times 60$
$= 30$ cm

2. segitiga
- Segitiga sama sisi
- Segitiga sama kaki
- segitiga siku-siku

Segiempat
- Persegi
- Persegi Panjang
- layang - layang
- Belah ketupat

3. $L_{bangunan} = 34 \times 34$
$= 1165$ m
$L_{atap} = \frac{1}{2} \times 38 \times 9$
$= \frac{1}{2} \times 342$
$= 171$

Figure 2 The Answer Sheet of Student EM

A closer examination of this answer sheet provides a more detailed picture of the pattern of students' obstacles. In item 1, the student was able to write the triangle area formula ($A = ½ \times$ base $\times$ height) and correctly substitute the values of the base and height, but an error occurred at the final stage of calculation, indicating that the student had not yet been fully careful in basic arithmetic operations despite having mastered the formula's structure. In item 2, the student was able to name various types of triangles (equilateral, isosceles, right-angled) and types of quadrilaterals (square, rectangle, kite, rhombus) fairly well, indicating that declarative knowledge or memorized classification of plane figures had been relatively mastered. However, in item 3, which required the application of the area concept to a more complex figure, the student's answer showed a more significant error in the calculation process and in determining the correct unit for the final answer. This pattern suggests that the student's understanding of triangles and quadrilaterals remained procedural and memorization-based at the level of definitions and formulas, but was not yet fully coherent at the level of concept application, particularly when faced with more complex or non-routine problem variations.

Interview results with the student concerned reinforced this finding, and similar hesitancy was also observed among other interviewed students, indicating that this psychological factor was not isolated to a single case. Classroom observation similarly revealed the same hesitancy when the student was asked to explain the reasoning behind the steps taken. This indicates that the obstacle experienced was not purely cognitive but was also colored by a psychological factor, namely a lack of confidence when facing computational problems.

Overall, observation and interview results across several students showed that the learning obstacles experienced could be categorized into three types, consistent with the learning obstacle categorization proposed by Lutfi, Juandi, and Jupri (2021): an ontogenic obstacle related to students' mental and cognitive readiness to grasp the concepts of triangles and quadrilaterals appropriate to their developmental stage; an epistemological obstacle related to students' limited understanding of the underlying geometric concepts themselves, such as errors in distinguishing base and height when the orientation of a plane figure was changed; and a didactical obstacle stemming from conventional teaching approaches that provided insufficient opportunity for students to independently construct conceptual understanding through contextual experience.

3.1.3 Didactic Design of Instruction

Based on the analysis of these three types of learning obstacles, the didactic design developed in this study was focused on addressing three key issues: students' errors in distinguishing elements of plane figures such as base and height, misconceptions in applying the area formula to varied problem types, and low active student engagement in the learning process due to insufficiently contextual teaching methods. The didactic design developed was grounded in the principle of green numeracy-based instruction, an approach that deliberately connects mathematical concepts with real-life contexts and environmental issues around students, such as land-area measurement, roof-shape observation, and building floor-area calculation.

This design was realized in the form of an interactive companion book or Student Worksheet (LKPD) developed for four sessions, with each session focused on one core sub-concept of triangles and quadrilaterals, ranging from an introduction to the elements of plane figures, to the application of perimeter and area formulas, to the application of these concepts within real environmental contexts. The instructional structure within the LKPD was designed to be student-centered, with a learning sequence beginning with a "let's observe" activity to spark students' curiosity about the environmental phenomenon that would serve as context, followed by a "let's read" activity to introduce the relevant mathematical concept, then a guessing activity that encouraged students to formulate an initial conjecture before performing the actual measurement or calculation, and concluding with a practice activity to reinforce the conceptual understanding that had been constructed.

3.1.4 Implementation

The implementation of the didactic design was carried out in two main stages. The first stage consisted of testing for learning obstacles in the higher grade, as previously described in the section on students' learning obstacles, while the second stage consisted of implementing the companion book in the lower grade, involving 33 students, under the guidance of the mathematics teacher. The implementation process took place over three sessions, following the stages designed within the LKPD. In the first session, students were introduced to an environmental observation activity around the school to identify shapes of triangles and quadrilaterals present in the surrounding environment, such as building roof shapes and land plots. During the second session, students made direct measurements of these objects using basic measuring tools before calculating their area and perimeter according to their measurements. In the third session, students engaged in reflective group discussions to compare measurement and calculation results across groups, and presented their findings to the class.

Observations across these three sessions revealed a fairly significant improvement in students' conceptual understanding. Students were able to re-explain the relationship among base, height, and the area of a triangle using their own words, rather than merely repeating memorized formulas, and were able to identify the elements of quadrilaterals without confusion even when the shapes were presented in rotated or unusual orientations—previously one of the main sources of obstacle. Learning activities oriented toward direct experience also made students appear more confident and active in expressing opinions and asking questions, in contrast to their initially passive condition. Overall, instruction that had initially been theoretical and teacher-centered shifted into activities that were more participatory, collaborative, and oriented toward students' real experiences.

3.1.5 Analysis of Implementation Results

The analysis of the implementation process revealed four main outcomes. First, an improvement in conceptual understanding occurred, evident from students' ability to independently explain relationships among elements of plane figures and apply them to real measurement contexts without the confusion previously observed. Second, an increase in student engagement and learning activity occurred, evident from students' enthusiasm in conducting direct measurements, engaging in group discussions, and presenting their findings, compared to their initially passive and less participatory condition. Third, a positive shift in students' attitudes and responses toward the applied instructional design occurred, with students reporting that the learning became more enjoyable and meaningful because it connected mathematical concepts to real activities in their surrounding environment. Fourth, preliminary results obtained from observation and reflection indicated an overall positive trend regarding the effectiveness of the green numeracy-based didactic design in addressing students' learning obstacles in triangles and quadrilaterals, although some residual difficulties remained, such as some students' confusion between height and the hypotenuse of a triangle, as well as difficulties in performing accurate field measurements.

3.2 Discussion

The main findings of this study reveal two interrelated aspects. First, prior to the intervention, students experienced three types of learning obstacles in the topic of triangles and quadrilaterals (ontogenic, epistemological, and didactical) reflected in students' difficulty distinguishing base and height, errors in applying the area formula to more complex problems, and passive attitudes coupled with low confidence when facing computational problems. Second, following the implementation of the green numeracy-based didactic design, significant improvements occurred in conceptual understanding, learning engagement, and student confidence.

The first finding, regarding students' learning obstacles, can be explained through a combination of observation and written test results. Students' answer sheets showed that they were able to write formulas and classify types of plane figures reasonably well, but made errors when these formulas needed to be applied to more complex or non-routine problems. This pattern indicates a gap between procedural and conceptual knowledge, a phenomenon widely reported in the mathematics education literature.

This finding was further confirmed by interview results. A student with the initials EM interviewed also admitted that he regularly feels tentative and unsure of himself when confronted with computational questions, even though he had memorized, verbatim, the formula presented in class. This comment suggests that the learning difficulty encountered was not a purely cognitive one, but emotionally charged too. The same conclusion as Hendriyanto et al. (2024) challenges epistemological and psychological with students are often causally linked, resulting from flawed knowledge transposition at all levels; from design for curriculum to practice in the classroom.

As to the second finding on understanding and engagement after employing the green numeracy-based didactic design, this can be understood in light of both observation data collected during three implementation sessions as well as students' reflections after instruction. When students were guided to conduct direct measurements of real objects such as building roofs and land plots, they no longer relied solely on memorized formulas but constructed understanding through concrete experience, which was subsequently reflected upon verbally in group discussions. This finding was further confirmed by interview results after implementation, in which the students stated that this instructional design was more engaging because it involved hands-on activities within their environment; and found doing so made understanding this concept much easier since they could relate to things that comprised their daily lives. This finding is also in line with the results of Delamontano et al. (2023) and the findings of Boehm et al. A similar pattern was reported in the study of Vale and Barbosa (2023), who highlight that it is through active learning strategies involving physical and social engagement with other students—and not cognitive activity in the classroom—that mathematics anxiety is alleviated, and thus an increase in learning engagement may be obtained as per how this didactic design successfully transformed students' attitudes from passive to more interactive and confident learners after its implementation.

Together, these findings lend support to a couple of theories in mathematics education. In the first place, the result regarding catching a tension by developing a design driven from learning obstruction studies is aligned with an important aspect of Theory of Didactical Situations, as stated in metasynthesis study which restates its core: learning situations must be especially built to cause cognitive conflict for students (Hortelano & Prudente 2024), corroborated also by that geometry instruction without deliberately designed didactical situation fails to help one build deep understanding as Genc and Ergan (2022) highlighted. Second, the finding that direct experience and real environmental context instruction is effective supports the principle of constructivism in mathematics education, which states that students build knowledge from concrete experiences as opposed to simply transferring information by mere conventional methods from teacher input to student output (Vale & Barbosa, 2023) with QN-DL active learning strategies working simultaneously on cognitive, social and physical aspects of students. Hence, the results indicate that combining either a DDR-based analysis on obstacles with a contextual level like green numeracy can be an appropriate solution for bridging the procedural and conceptual gap and also promoting attitudes and environmental awareness.

4. Conclusions

The findings of this study point to two central conclusions. Students were found to experience three types of learning obstacles in the topic of triangles and quadrilaterals (ontogenic, epistemological, and didactical), reflected in a gap between procedural and conceptual knowledge as well as a lack of confidence when facing computational problems. Furthermore, the green numeracy-based didactic design developed through the Didactical Design Research approach proved effective in addressing these obstacles, as indicated by improvements in conceptual understanding, learning engagement, and student confidence following instruction linked to real measurement activities in the surrounding environment. This study concludes that students experienced three types of learning obstacles in the topic of triangles and quadrilaterals (ontogenic, epistemological, and didactical) reflected in a gap between procedural and conceptual knowledge as well as students' lack of confidence when facing computational problems, and that the green numeracy-based didactic design developed through the Didactical Design Research approach proved effective in addressing these obstacles, as indicated by improvements in conceptual understanding, learning engagement, and student confidence following instruction linked to real measurement activities in the surrounding environment. Nevertheless, this study has several limitations, including a participant pool limited to a single madrasah, meaning the findings cannot yet be broadly generalized; a relatively short implementation duration of only three sessions, which does not yet capture the long-term impact on conceptual retention; and the persistence of residual difficulties, such as students' confusion between height and the hypotenuse of a triangle and inaccuracies in field measurement, indicating that the developed didactic design has not yet fully resolved every aspect of students' epistemological obstacles. Therefore, future researchers are recommended to expand the scope of participants across several schools with differing characteristics to test the generalizability of the findings, extend the implementation duration and conduct follow-up measurements (delayed post-tests) to evaluate the sustainability of conceptual understanding, develop additional activities specifically aimed at training measurement accuracy and the distinction between height and hypotenuse in irregular plane figures, and consider integrating technology such as GeoGebra to strengthen conceptual visualization while expanding the variety of green numeracy contexts beyond land and building measurement (for instance, to issues of waste management or energy efficiency) so that the environmental awareness fostered becomes more comprehensive.

Recommendations

Based on the findings, the following recommendations are made: teachers should integrate real-world environmental measurement activities, such as observing roof shapes and measuring land plots, into triangle and quadrilateral instruction to help students construct conceptual understanding rather than rely on memorized formulas. Schools should provide opportunities and adequate facilities for teachers to design and implement contextual, green numeracy-based learning materials rather than relying solely on conventional lecture-based instruction. Curriculum developers and textbook authors should incorporate environmentally contextualized tasks into geometry materials to strengthen the connection between abstract mathematical concepts and students' everyday experiences. For future research, it is recommended that subsequent studies expand the participant pool across multiple schools with differing characteristics, extend the implementation duration with delayed post-tests to assess the long-term retention of conceptual understanding, develop additional activities specifically targeting measurement accuracy and the distinction between height and hypotenuse in irregular plane figures, integrate technology such as GeoGebra to strengthen conceptual visualization, and broaden the variety of green numeracy contexts beyond land and building measurement to include issues such as waste management or energy efficiency.

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Original Article

Declarations

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