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Original Article

Redesigning number pattern instruction: A didactical design grounded in learning obstacle analysis for junior high school students

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Abstract

The concept of number patterns is an essential prerequisite for arithmetic sequences and series, yet a preliminary study of 15 eighth-grade students at SMP Islam Riyadhussolihin NW Bukit Tengger revealed that most students still experienced learning obstacles when solving problems on even-odd, triangular, square, and rectangular number patterns. This study aimed to identify the types of learning obstacles experienced by students, to design a hypothetical didactical design grounded in these findings, and to examine its effectiveness through classroom implementation. A qualitative approach using Didactical Design Research (DDR)—comprising prospective, metapedadidactic, and retrospective analysis—was employed. Data were collected through a learning-obstacle diagnostic test, classroom observation, and semi-structured interviews with selected students, then analysed using source and method triangulation. The findings revealed that the dominant obstacle was ontogenic-psychological in nature, reflected in students' carelessness and haste in completing tasks, alongside conceptual obstacles related to the prerequisite concept of even and odd numbers. Based on these findings, a Hypothetical Learning Trajectory (HLT) comprising four sequential didactical situations, anticipated student responses, didactical-pedagogical anticipations, and a Student Worksheet (LKS) was developed. Implementation of the HLT reduced learning obstacles, although new difficulties emerged and were subsequently accommodated through a revised didactical design. This study contributes to enriching Didactical Design Research literature on number patterns while producing a classroom-ready instructional package for secondary mathematics teachers.

Keywords: Didactical Design; Learning Obstacle; Number Pattern; Didactical Design Research

1. Introduction

The ability to reason inductively through pattern recognition is a foundational component of mathematical thinking required by secondary school mathematics curricula, as the concept of number patterns serves as a conceptual bridge toward arithmetic and geometric sequences and series in subsequent grades. This ability is also an indicator of computational thinking, as emphasized by Helsa et al. (2023), who argue that pattern recognition is one of five computational-thinking indicators that should be cultivated

from the earliest stages of number-pattern instruction in junior high school. Unfortunately, this urgency has not been fully matched by students' actual learning outcomes in the field.

A range of recent studies continues to show that students face systematic difficulties in understanding number patterns. Sartika et al. (2022) found that junior high school students tend to memorize the nth-term formula without understanding the underlying structure of the pattern, making them prone to error when confronted with contexts that differ from previously studied examples. Similarly, Hasibuan et al. (2023) reported that most junior high school students had only reached the stage of guessing a pattern without being able to validate and generalize the corresponding formula, indicating weak inductive reasoning on this topic. Arigawati & Kusnandi (2021) further found a significant difference in critical-thinking ability between students with field-dependent and field-independent cognitive styles when solving number-pattern problems, underscoring that learning obstacles on this topic are multidimensional rather than merely procedural in nature.

A comparable phenomenon emerged in the preliminary study conducted for this research. Of 15 eighth-grade students at SMP Islam Riyadhussolihin NW Bukit Tengger who completed a learning-obstacle diagnostic test, the majority exhibited recurring errors when determining the nth term of even, odd, triangular, square, and rectangular number patterns. These errors were not merely procedural (misapplication of a formula) but also conceptual—for instance, confusion in distinguishing the characteristics of even and odd numbers, a concept that should already have been mastered in elementary school. Such a difficulty, within the framework of the theory of didactical situations, is known as a *learning obstacle*, that is, a barrier that prevents students from attaining a coherent conceptual understanding (Chevallard & Bosch, 2020a).

Learning obstacles are commonly classified into three categories: ontogenic obstacles (related to students' mental readiness and developmental stage), didactical obstacles (arising from teachers' selection of instructional materials and teaching strategies), and epistemological obstacles (stemming from the limited contexts in which the concept itself has been encountered). These three categories are closely intertwined and, if not anticipated from the planning stage, tend to recur in subsequent lessons (Hendriyanto et al., 2024). Fuadiah & Suryadi (2019), in their study on negative integers, similarly showed that student misconceptions left unanticipated by the teacher persist and hinder understanding of subsequent material. This finding reinforces the argument that a systematic identification of learning obstacles is an essential—not merely supplementary—step prior to designing instruction.

Efforts to address learning obstacles through instructional design are not new. The Realistic Mathematics Education (RME) approach, for instance, has widely been used to design learning trajectories for the topic of number patterns. Fauzan & Diana (2020) developed an RME-based learning trajectory for number patterns in grade VIII that proved valid in terms of content and construct and was shown to enhance students' reasoning. Susanta et al. (2023) likewise developed number-pattern mathematical-literacy tasks set in the local context of Bengkulu, while Umasugi et al. (2022) designed a social-arithmetic learning trajectory grounded in the culinary context of Yogyakarta. These studies confirm that carefully designed realistic contexts and learning trajectories can substantially improve the quality of mathematics instruction.

Nevertheless, most of the aforementioned learning-trajectory studies on number patterns were designed on the basis of curriculum analysis and theoretical review, without being preceded by an in-depth empirical identification of learning obstacles in a specific school context. On the other hand, Didactical Design Research (DDR) studies that do depart from empirical learning-obstacle analysis—such as Pauji et al. (2023) on the concept of number, and Jamilah et al. (2024) on the volume of a cuboid—generally focus on topics other than number patterns at the junior high school level. To the best of the authors' knowledge,

studies that specifically integrate learning-obstacle analysis, the development of a Hypothetical Learning Trajectory (HLT), and retrospective testing on the concept of number patterns in grade VIII—particularly within pesantren-based (Islamic boarding school) institutions whose student characteristics remain comparatively under-researched—are still limited. This gap underlies the need for the present study.

Based on the foregoing background, this study was designed to answer three research questions:

1) what types of learning obstacles do eighth-grade students experience in learning the concept of number patterns?
2) what hypothetical didactical design can be developed to anticipate these learning obstacles?
3) how effective is the implementation of this didactical design, as evaluated through metapedadidactic and retrospective analysis?

2. Method

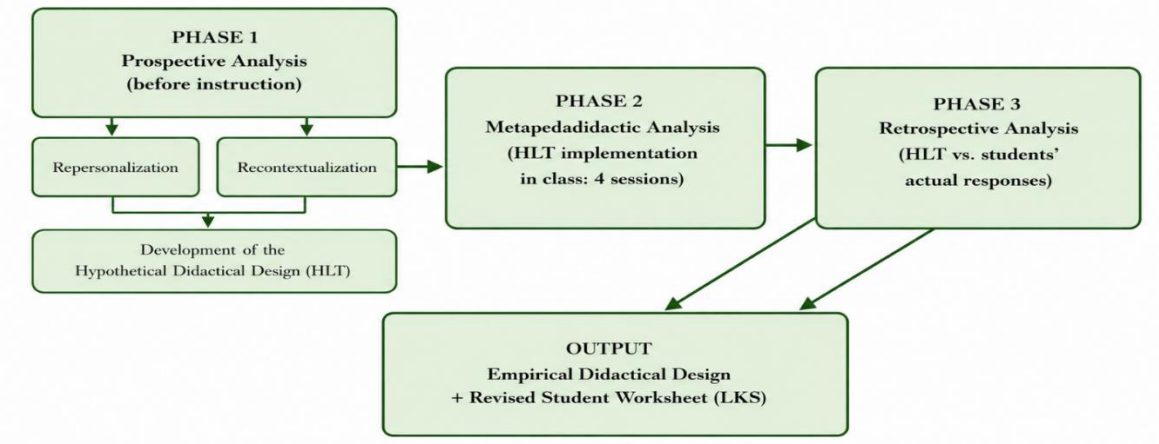
2.1 Research Design

This study employed a qualitative approach using Didactical Design Research (DDR), a research design intended to construct didactical situations based on learning-obstacle analysis so as to produce a learning trajectory capable of reducing students' learning obstacles (Hidayat & Suryadi, 2016). DDR was selected because its cyclical and reflective character allows the researcher not only to diagnose learning obstacles but also to test and revise the instructional design based on empirical data from actual classroom implementation, consistent with the principle of praxeological-didactical analysis proposed by Yunianta et al. (2023) for evaluating mathematics instructional materials.

As illustrated in Figure 1, this study was conducted through three sequential and interrelated phases. The first phase, prospective analysis, comprised repersonalization (analysis of textbooks and teacher-prepared materials), recontextualization (analysis of learning obstacles through initial tests and interviews), and the development of a Hypothetical Learning Trajectory (HLT). The second phase, metapedadidactic analysis, was carried out by implementing the HLT across four classroom sessions while recording students' actual responses. The third phase, retrospective analysis, compared the anticipated responses in the HLT with students' actual responses to produce a revised, empirically grounded didactical design, in line with the anthropological theory of the didactic, which emphasizes the interrelation between teaching practice and the structure of the knowledge being constructed (Suryadi et al., 2023).

Figure 1

Research Design Flow (Didactical Design Research)



2.2 Research Participants

Participants were determined through purposive sampling, that is, the selection of subjects based on specific considerations aligned with the research objectives rather than statistical generalization. The primary subjects were 15 eighth-grade students at SMP Islam Riyadhussolihin NW Bukit Tengger who had received instruction on number patterns and who took part in the learning-obstacle diagnostic test. From these 15 students, 3 students whose error patterns were representative were selected for in-depth interviews to confirm the types of learning obstacles identified from their written answers. One mathematics teacher responsible for the class was also involved as a triangulation source, particularly to confirm the appropriateness of the instructional materials and teaching strategies previously used.

Grade VIII students were selected on the consideration that number patterns are formally taught at this level according to the structure of the junior high school mathematics curriculum, so that the learning obstacles identified reflect an ongoing instructional condition rather than material not yet taught. A summary of participant characteristics is presented in Table 1.

Table 1

Characteristics of Research Participants

No	Participant Group	Number	Sampling Technique	Role in the Study
1	Grade VIII students (learning-obstacle test)	15 students	Purposive sampling	Source of diagnostic-test data
2	Selected students (in-depth interview)	3 students	Purposive sampling based on error patterns	Confirmation of learning-obstacle type
3	Grade VIII mathematics teacher	1 teacher	Direct designation (class teacher)	Source and contextual triangulation

2.3 Data Collection

Data were collected through three complementary techniques: a written test, observation, and semi-structured interviews. The learning-obstacle diagnostic test, consisting of open-ended number-pattern items, was used to map students' error patterns in writing. Classroom observation was conducted during every HLT-implementation session to record students' interaction with the didactical situations presented, including verbal and non-verbal responses not always captured in written answers. Semi-structured interviews were used to trace the reasoning behind students' errors that could not be inferred from written work alone, consistent with the principle that both teachers' and researchers' interviewing strategies play a central role in accurately diagnosing learning obstacles.

All three instruments were developed based on a blueprint referring to indicators of conceptual understanding of number patterns (even-odd, triangular, square, rectangular) and were content-validated by mathematics education lecturers prior to use. Details of the instruments, their purposes, and the resulting data are presented in Table 2.

Table 2

Data-Collection Instruments and Techniques

No	Instrument	Purpose	Type of Data	Timing
1	Learning-obstacle diagnostic test	Mapping types and patterns of student errors on number patterns	Written answer sheets of 15 students	Preliminary study
2	Classroom observation sheet	Recording student responses during HLT implementation	Field notes and classroom-activity recordings	4 implementation sessions
3	Semi-structured interview guide	Confirming the causes of student and teacher difficulties	Interview transcripts	Preliminary study and post-implementation

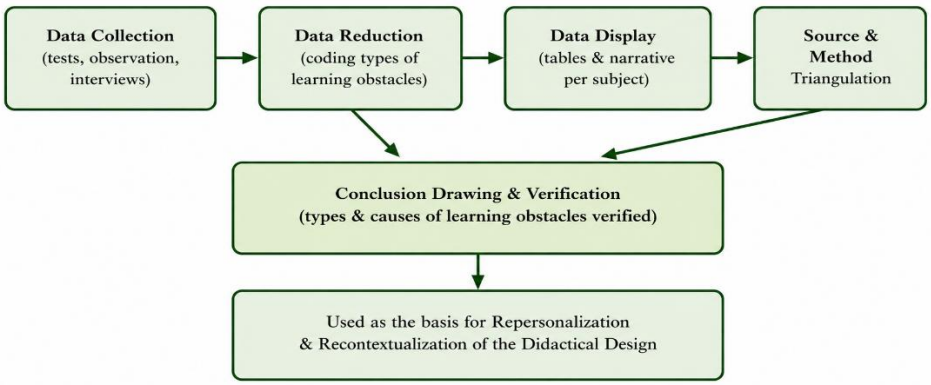
2.4 Data Analysis

Data analysis followed the interactive qualitative model comprising data reduction, data display, and conclusion drawing/verification, carried out concurrently with data collection. Test, observation, and interview data were first reduced by categorizing student errors into types of learning obstacles (ontogenic, didactical, epistemological) following a thematic coding framework as described by (Kiger & Varpio, 2020), before being presented in tabular form and descriptive narratives per subject.

Data trustworthiness was established through source triangulation (students, teacher, instructional documents) and method triangulation (test, observation, interview), as illustrated in Figure 2. The results of triangulation at the prospective-analysis stage informed the repersonalization and recontextualization of the didactical design; the results of the metapedadidactic and retrospective analyses were used to evaluate the correspondence between the anticipated responses in the HLT and students' actual responses, thereby producing a revised, empirically grounded didactical design.

Figure 2

Data Analysis Process



2.5 Ethical Clearance

This study was conducted in accordance with the ethical principles of educational research. Research permission was obtained from the school (the principal and the subject teacher) prior to data collection. All student and teacher participants were informed of the study's purpose and voluntarily agreed to participate (informed consent), confirmed verbally and acknowledged by the homeroom teacher, given the participants' status as students under the auspices of an educational institution. Student identities in the reporting of results were anonymized using codes (Student 1, Student 2, Student 3) to preserve participants'

confidentiality and dignity. Research data were stored and used solely for academic purposes and were not disseminated without permission.

3. Results

3.1 Results of Learning Obstacle Analysis on the Number Pattern Concept

The learning-obstacle diagnostic test administered to the 15 grade VIII students revealed recurring error patterns across three main aspects: (1) application of the nth-term formula for odd number patterns, (2) classification of even and odd number patterns, and (3) application of the formula for rectangular number patterns. Representative excerpts from students' answer sheets are presented in Table 3.

Table 3

Analysis of Students' Answer Sheets

No	Excerpt of Student Answer	Remarks
1	Determining U6 of the pattern 1, 3, 5, 7 using the formula Un = 2n − 1, the student wrote U6 = 2.6 − 1 = 2.5 = 10 (incorrect order of operations)	Indication of an unmastered prerequisite—order of operations (ontogenic-conceptual obstacle)
2	Classifying the sequence 10, 12, 14, 16, 18 as an odd number pattern	Indication of an unmastered prerequisite—characteristics of even and odd numbers (ontogenic-conceptual obstacle)
3	Writing the formula for the rectangular number pattern as Un = n^2 (confused with the formula for the square number pattern)	Indication of a procedural error in distinguishing formulas across patterns (didactical obstacle)

Based on Table 3, several possible causes of students' learning obstacles were identified. To confirm these indications more thoroughly—since written errors alone do not necessarily reflect the true underlying cause of students' difficulties—interviews were conducted with three selected students. This approach is consistent with Prabowo (2022)'s argument that interview triangulation is required to ensure the credibility of learning-obstacle findings, given that superficially similar errors on written answer sheets may stem from different underlying causes. This triangulation approach also aligns with the view of Kiger & Varpio (2020) that qualitative thematic analysis requires cross-verification across data sources to ensure that the resulting categories genuinely represent the phenomenon under investigation.

An interview with Student 1 was conducted to trace the cause of the error on item 1. The interview excerpt is presented in Table 4.

Table 4

Interview Excerpt with Student 1

Researcher's Question	Student's Answer
Why did you answer 2x6−1 = 2x5?	"Because 6−1 = 5."
Have you learned addition, subtraction, multiplication, and division before?	"Yes, I have."
Usually, for a problem like that, how do you solve it?	"You multiply 2x6 first, then subtract 1."
So you actually know how to do it?	"Yes, I do."
Then why did you answer it that way?	"I don't know."

> *"I actually knew the correct order of operations, but when I was answering, I made a mistake without realizing it." — Student 1, post-test interview transcript.*

Student 1's response in Table 4 reveals a gap between the procedural knowledge actually possessed ("you multiply 2x6 first, then subtract 1") and the erroneous written work produced. This gap indicates that the error did not stem from a lack of conceptual knowledge but rather from carelessness and haste in completing the task—a type of obstacle that, within the classification proposed by (Chevallard & Bosch, 2020b), falls under the ontogenic-psychological category. This phenomenon—a discrepancy between genuinely mastered procedural knowledge and its erroneous application during task completion—is also consistent with the concept of *mental set* proposed by (DeCaro, 2016), namely students' tendency to apply a procedure automatically and hastily without checking its fit with the structure of the problem, so that the procedural flexibility and conceptual understanding they genuinely possess are not optimally utilized.

An interview with Student 2 focused on the misclassification of even and odd numbers, as presented in Table 5.

Table 5. Interview Excerpt with Student 2

Researcher's Question	Student's Answer
Is it true that 10, 12, 14, 16, 18 are odd numbers?	"I think that's correct."
What about 1, 5, 9, 11—what kind of numbers are those?	"Even numbers. Is my answer wrong?"
It could be said that way.	"Oh, I see. So what is the correct answer?"
2, 4, 6, 8 are even numbers, and 1, 3, 5, 7 are odd numbers. Please remember that.	"Oh, okay."
When you previously learned about even and odd numbers, did you not fully understand it?	"Yes, that's right."
Why is that?	"Because I was sick, and when I came back to school there was a test right away."

Student 2's admission regarding her absence when the prerequisite material was taught indicates that the obstacle she experienced was ontogenic-conceptual in nature—that is, conceptual unreadiness resulting from a gap in prior learning experience—rather than mere carelessness while solving the problem. This finding is consistent with Fuadiah & Suryadi (2019)'s argument that misconceptions left uncorrected in

prerequisite material will continue to be carried forward and hinder understanding of material that is structurally dependent on it.

An interview with Student 3 traced the cause of the misapplication of the rectangular number-pattern formula, as presented in Table 6.

Table 6

Interview Excerpt with Student 3

Researcher's Question	Student's Answer
Is it true that the formula for the rectangular number pattern is n^2?	"Yes, I think so."
Are you sure that's the formula?	"I don't know."
Why did you answer that way?	"My friends had already finished and left, Miss."
So you answered that way because you were in a hurry?	"Yes, Miss. Sorry."

The three interview excerpts above consistently point to two dominant categories of learning obstacle: ontogenic-psychological obstacles (carelessness and haste, experienced by Students 1 and 3) and ontogenic-conceptual obstacles (a gap in mastery of prerequisite material, experienced by Student 2). No strong indication of an epistemological obstacle was found in this study, given that students were essentially able to explain the correct procedure when asked to reflect on it again by the researcher. This pattern of findings is consistent with Sartika et al. (2022), who likewise found a predominance of ontogenic obstacles in number-pattern instruction at the junior high school level.

3.2 Description of the Hypothetical Didactical Design

Based on the learning-obstacle analysis presented in Section 3.1, a hypothetical didactical design was developed that explicitly anticipates both identified categories of obstacle. Didactical-pedagogical anticipation was designed not merely as answer correction, but as a didactical situation that prompts students to re-examine their own thinking process. A summary of the proposed solutions is presented in Table 7.

Table 7

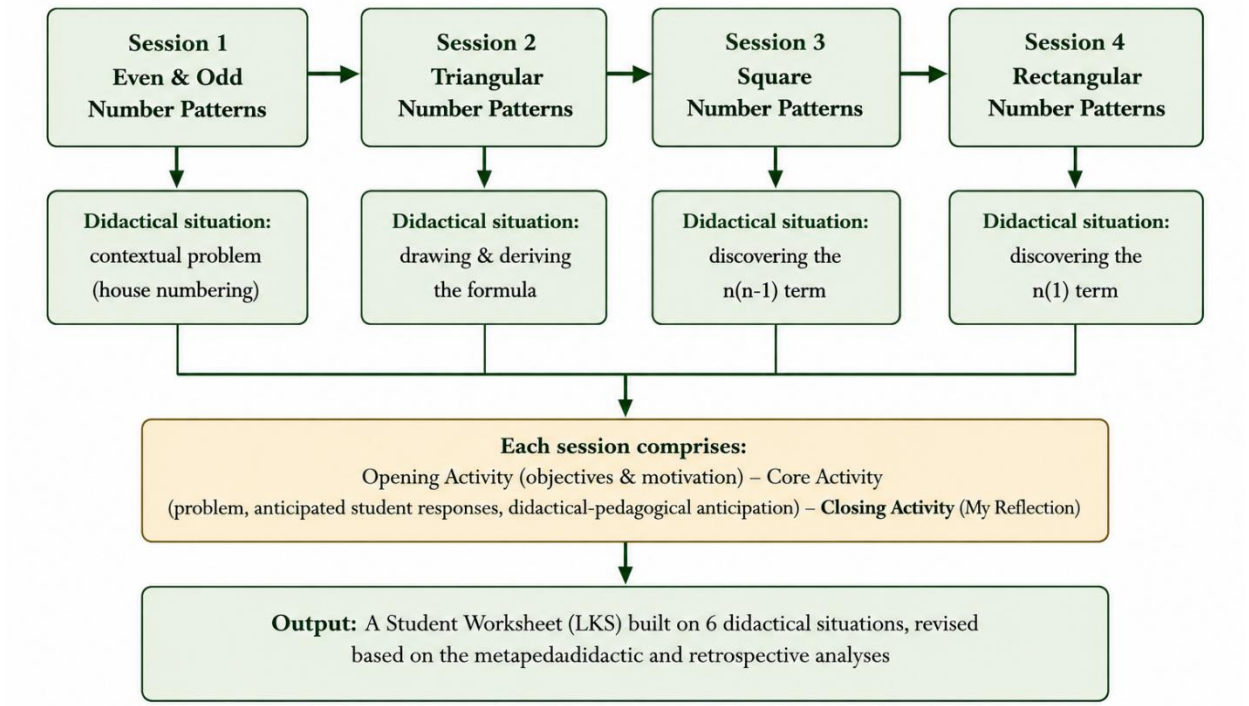
Alternative Solutions in the Didactical Design

No	Learning Obstacle/Cause	Alternative Solution	Realization in the Didactical Design
1	Students' carelessness/haste (ontogenic-psychological)	Asking students to check their answers before submission	An explicit prompt before each activity: "Check your answer again before submitting it," included on every worksheet
2	Gap in mastery of the even-odd prerequisite (ontogenic-conceptual)	Reactivating prerequisite knowledge through a contextual problem before entering the core material	An opening didactical situation in Session 1 using a contextual house-numbering problem
3	Confusion between formulas across patterns (didactical-procedural)	Directly comparing visual representations across patterns before introducing formal formulas	Drawing-and-comparison activities in Sessions 2–4 prior to formula generalization

The alternative solutions in Table 7 were subsequently elaborated into a Hypothetical Learning Trajectory (HLT) consisting of four sequential sessions. The HLT structure was designed so that each session comprises an opening activity (delivery of learning objectives and motivation), a core activity (presentation of a contextual problem, anticipated student responses together with their anticipations, and group-based problem-solving activities), and a closing activity in the form of student self-reflection ("My Reflection"). The overall HLT structure is presented in Figure 3.

Figure 3

Structure of the Hypothetical Learning Trajectory (HLT) for the Number Pattern Concept



The first session focused on reinforcing the prerequisite material—even and odd number patterns—through a contextual problem involving house numbering in a village. Students were asked to determine a house number and its ordinal term based on a given even-number pattern. This design specifically anticipates the ontogenic-conceptual obstacle experienced by Student 2, by placing the re-introduction of even-odd number characteristics as a foundation before students proceed to more complex number patterns, in line with the principle of Realistic Mathematics Education that instruction should begin from contexts close to students' own experience (Van den Heuvel-Panhuizen & Drijvers, 2020).

The second session introduced the triangular number pattern through an activity of drawing arrangements of dots and independently discovering the regularity of term increments, before the formula $Un = \frac{1}{2}n(n+1)$ was formalized. The third and fourth sessions addressed the square and rectangular number patterns respectively, through tiered problems given across groups (for example, determining U6 and U9 for the same pattern), so that students gained direct experience comparing the two formulas—$Un = n^2$ for the square pattern and $Un = n(n+1)$ for the rectangular pattern—side by side. This strategy of comparing visual representations before formalizing the corresponding formula was designed to anticipate the procedural confusion experienced by Student 3, consistent with the findings of Sutarni et al. (2024) that RME-based worksheets incorporating a concrete-to-abstract representational sequence are more effective in improving students' mathematical ability than the direct presentation of formulas. This principle of

instructional-material development is also consistent with Pertiwi et al. (2020), who emphasized that the development of mathematics instructional materials should be responsively tailored to students' actual learning conditions, and with the RME-based social-arithmetic learning trajectory developed by Fauzan et al. (2018), which likewise demonstrated that sequencing activities from real-world context toward formal representation can be consistently applied across various secondary mathematics subtopics.

3.3 Results of Metapedadidactic Analysis

Implementation of the HLT across the four sessions showed that most student responses corresponded to the anticipations formulated at the design stage; however, several new difficulties emerged during the learning process itself, requiring additional on-the-spot anticipation by the teacher. In Session 1, although the contextual didactical situation (house numbering) successfully stimulated students to recognize the pattern intuitively, a number of students still struggled to explicitly distinguish the characteristics of even and odd number patterns when asked to justify their answers—indicating that reinforcing the prerequisite concept requires more than a single activity.

In Session 2, a new difficulty arose when students were asked to apply the triangular-number-pattern formula ($U_n = \frac{1}{2}n(n+1)$) to larger term values; most students were able to draw the pattern visually but had not yet been able to independently translate this visual pattern into an algebraic formula. In Session 3, students generally solved square-number-pattern problems well; however, in Session 4 a number of students again struggled to distinguish square from rectangular number patterns when both types of problems were presented side by side in a single session—confirming that the didactical-procedural obstacle identified in Table 3 had not been fully resolved through a single learning cycle.

Overall, the initial didactical design proved reasonably effective in reducing the intensity of errors compared with the condition observed during the preliminary study; however, the emergence of these new difficulties provided an important basis for the subsequent retrospective analysis to yield a revised didactical design, consistent with the cyclical nature of DDR, which emphasizes that a didactical design is always provisional and continuously refined based on implementation evidence (Jamilah et al., 2024).

3.4 Results of Retrospective Analysis and the Revised Didactical Design

Retrospective analysis, which links the results of prospective and metapedadidactic analysis, produced a number of concrete revisions to the initial didactical design. In Session 1, a new task was added along with expanded instructions and a broader set of anticipated responses and anticipations, given that students' difficulty distinguishing even-odd characteristics persisted through the initial implementation. In Session 2, a task requiring students to independently draw the triangular number pattern was added as an additional bridge from visual representation to algebraic representation before students were asked to state the formula. In Sessions 3 and 4, the wording of the problems was revised, instructions were sharpened, and the range of anticipated responses and anticipations was broadened to reduce confusion between the square and rectangular patterns.

These refinements produced an empirically grounded didactical design that had been tested through one complete cycle of prospective, metapedadidactic, and retrospective analysis. This revised didactical design, together with its accompanying Student Worksheet (LKS), constitutes the principal output of the study and has the potential to be used directly by mathematics teachers in schools with relatively similar student characteristics, as similarly recommended by Meika et al. (2025), who found that an e-didactical design refined through a learning-obstacle-based revision cycle was able to minimize students' learning obstacles in subsequent implementation.

4. Discussion

The findings of this study both corroborate and extend previous research on learning obstacles related to number patterns. The predominance of ontogenic-psychological obstacles (carelessness, haste) found in Students 1 and 3 is consistent with Sartika et al. (2022)'s conclusion that junior high school students' errors on the topic of number patterns stem more from weaknesses in validating their own answers than from a genuine lack of conceptual understanding. Nevertheless, this study also found that some obstacles were ontogenic-conceptual in nature, resulting from an individual gap in learning experience (as in the case of Student 2, who was absent when the prerequisite material was taught)—a nuance not yet widely foregrounded in prior DDR literature on number patterns. This finding illustrates that learning obstacles on the same topic may stem from a combination of cognitive and individual situational factors, meaning that an effective didactical design must accommodate both simultaneously.

From the perspective of instructional design, the results of this study align with a range of studies on the effectiveness of context-based Hypothetical Learning Trajectories (HLTs). Fauzan & Diana (2020) showed that an RME-based HLT on number patterns enhanced grade VIII students' reasoning, while Prahmana (2016) emphasized that a hypothetical learning trajectory developed through research-based learning provides teachers with a clear framework for anticipating student responses. The present study reinforces this argument by showing that an HLT explicitly constructed from empirical learning-obstacle analysis—rather than curriculum analysis alone—is more precisely targeted at anticipating the specific difficulties experienced by students in a particular school context.

The strategy of comparing visual representations prior to formal-formula generalization, applied in Sessions 2–4, is likewise consistent with Larson (2017)'s findings in the context of linear-transformation instruction, which showed that a learning trajectory building conceptual understanding through concrete representation before symbolic representation is more effective than direct formula introduction. Similarly, Bahamonde (2017) emphasized the importance of gradual mathematical modeling within a linear-algebra learning trajectory so that students do not merely memorize procedures. In the same vein, students' difficulty distinguishing the square and rectangular pattern formulas in Session 4 can be understood as an incomplete knowledge transposition—as explained by Lundberg & Kilhamn (2018) regarding how students often struggle to transfer their understanding of proportionality across mathematical representations that appear structurally similar yet differ conceptually.

The finding that new difficulties continued to emerge during implementation, despite a didactical design carefully constructed from prior learning-obstacle analysis, underscores the essentially cyclical—rather than one-off—character of DDR. This is consistent with Hausberger (2017), who explained that didactical transposition—the process of transforming scholarly knowledge into knowledge that can be taught—invariably undergoes adjustment when confronted with the dynamic realities of the classroom. In a similar vein, Umasugi et al. (2022) likewise reported that retrospective analysis of a culinary-context social-arithmetic learning trajectory required more than a single revision cycle before learning obstacles could be consistently reduced, reinforcing the argument that a single HLT-implementation cycle is generally sufficient to reduce the intensity of obstacles but not necessarily to resolve them entirely.

Compared with approaches to learning-trajectory development based purely on curriculum analysis, such as Astuti & Wijaya (2020)'s work on the topic of sets, the empirical learning-obstacle-based approach used in this study offers the advantage of a closer alignment between didactical anticipation and students' actual needs in a specific classroom. Nevertheless, this approach also has limitations in terms of generalizability: the resulting didactical design is highly contextual to the characteristics of students at SMP Islam Riyadhussolihin NW Bukit Tengger, so its application in other schools requires adaptation based on locally conducted learning-obstacle analysis rather than direct replication. A further limitation lies in the

relatively small number of in-depth interview subjects (three students), meaning that the findings regarding the proportion of obstacle types should be interpreted qualitatively and exploratively rather than as a quantitative generalization to the wider student population.

From the perspective of technology integration, the potential further development of this didactical design might also consider incorporating interactive visual media. Fernández-Enríquez & Delgado-Martín (2020), for instance, showed that augmented reality as a supplementary learning resource can strengthen the visual representation of abstract mathematical concepts, which could plausibly be relevant for reinforcing the drawing activities for triangular and square number patterns in future research. More broadly, Garcia-Manrubia (2024) cautions that the effectiveness of a mathematics instructional design cannot be separated from students' affective factors, meaning that the carelessness and haste identified as the dominant cause of learning obstacles in this study should also be understood within a framework of self-regulation and self-confidence, rather than solely as a cognitive deficit.

Returning to the three research questions posed in the Introduction, the findings above may be summarized as follows. First, the types of learning obstacle experienced by grade VIII students in the number-pattern concept proved not to be singular, but rather a combination of ontogenic-psychological and ontogenic-conceptual obstacles, with no strong indication of epistemological or purely didactical obstacles. This finding differs somewhat from earlier studies—for instance Astriani et al. (2022), who found all three obstacle categories (ontogenic, epistemological, and didactical) represented in a relatively balanced manner on the topic of algebra—thereby showing that the profile of learning obstacles is topic-specific and context-specific rather than a universally applicable pattern across all secondary mathematics content.

Second, the resulting hypothetical didactical design shows that didactical-pedagogical anticipation is most effective when formulated as a learning situation that leads students to experience directly the consequences of their own carelessness (for example, through an explicit instruction to re-check their answers) and that provides a visual-representation bridge before symbolic representation. This approach is consistent with the principle of didactical phenomenology, which holds that a mathematical concept should ideally be introduced through a phenomenon that generates the need for that concept (Van den Heuvel-Panhuizen & Drijvers, 2020), rather than being presented as a stand-alone formal definition. Third, the results of the metapedadidactic and retrospective analyses demonstrate that the effectiveness of a didactical design is gradual and cumulative: the intensity of obstacles decreased markedly within a single implementation cycle, yet complete mastery across all number-pattern subtopics appears to require more than one revision cycle, consistent with the iterative character of DDR that has been widely emphasized in the literature Hidayat & Suryadi (2016).

The practical implications of these findings are significant for classroom mathematics teachers. Teachers cannot assume that student errors on number-pattern problems necessarily reflect a lack of conceptual understanding; as demonstrated in the cases of Students 1 and 3, students who genuinely possess the correct procedure may still make errors due to carelessness or haste. Diagnostic strategies based on reflective interviewing—as applied in this study—should therefore become a routine part of teachers' formative-assessment practice, rather than being confined to research contexts. Teachers should also remain alert to the possibility of students with individual learning-experience gaps (as in Student 2's case, resulting from absence), meaning that personalized remedial strategies—not classical repetition alone—remain necessary even when a classroom-level didactical design has been carefully constructed. This reinforces Contreras (2025)'s view that students' procedural and conceptual knowledge must be explicitly connected in instruction; as shown by the cases of Students 1 and 3, possessing correct procedural knowledge does

not automatically guarantee its consistent application without explicitly cultivated habits of self-checking within the didactical design.

At the curricular level, these findings also suggest that the sequencing of number-pattern material should explicitly incorporate a recap activity reinforcing the concepts of even and odd numbers before proceeding to more complex patterns, rather than assuming full mastery of elementary-school material. This recommendation is consistent with Hendriyanto et al. (2024)'s view that curriculum design which neglects learning-obstacle analysis on prerequisite material risks unknowingly carrying forward learning obstacles from earlier grades into subsequent ones. Accordingly, the contribution of this study extends beyond the single school in which it was conducted, offering a diagnostic-design framework that can be more broadly adapted in planning number-pattern instruction at the junior high school level.

5. Conclusion

This study yields three main conclusions. First, the prospective analysis revealed that the learning obstacles experienced by grade VIII students in the number-pattern concept were dominated by ontogenic-psychological obstacles (carelessness and haste in completing tasks) and ontogenic-conceptual obstacles arising from a gap in mastery of the even-odd number prerequisite, with no strong indication of epistemological obstacles. Second, based on these findings, a Hypothetical Learning Trajectory (HLT) was developed comprising four sequential didactical situations—even-odd, triangular, square, and rectangular number patterns—each incorporating anticipated student responses, didactical-pedagogical anticipations, and a Student Worksheet (LKS) explicitly designed to reduce both identified categories of obstacle. Third, the results of the metapedadidactic and retrospective analyses show that the initial didactical design was generally effective in reducing the intensity of learning obstacles, although new difficulties—particularly in applying the triangular-number-pattern formula and distinguishing the square from the rectangular pattern—continued to emerge and were successfully accommodated through a revised didactical design at the end of the research cycle.

This study offers three principal contributions. Theoretically, it enriches the body of Didactical Design Research by showing that learning obstacles on the number-pattern concept do not stem solely from cognitive-procedural factors, as widely reported in prior studies, but also from a combination of psychological factors and individual gaps in learning experience—a nuance that extends the classification framework of ontogenic learning obstacles in a more granular way within the context of number-pattern instruction. Practically, this study produces a classroom-ready instructional package in the form of an empirically grounded didactical design and an accompanying LKS, tested through one complete classroom-implementation cycle, which can be directly adapted by junior high school mathematics teachers, particularly in pesantren-based institutions whose student characteristics have relatively rarely been the focus of prior DDR research. Methodologically, this study demonstrates a complete, integrated cycle of prospective, metapedadidactic, and retrospective analysis on a single secondary-school mathematics topic, offering a procedural model that other researchers may replicate on different mathematical topics.

Future research is recommended to expand the number and variation of interview subjects to enrich the mapping of learning-obstacle types, to add further implementation cycles to fully resolve the new difficulties identified in this study, and to consider integrating interactive visual media into the didactical design for the number-pattern topic in schools with different student characteristics.

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Author Contribution

M.Z. contributed to the conceptualization, research design, data collection, data analysis, and preparation of the original manuscript. D.I. contributed to the development of the didactical design, data interpretation, and methodological refinement. M.G.I. contributed to the methodological framework, data analysis, and critical revision of the manuscript. A.A. contributed to the development and evaluation of the instructional design and interpretation of the findings. S.S. contributed to the theoretical framework, critical review, and refinement of the manuscript. R.J.S. contributed to data interpretation, manuscript review, and refinement of the final version. All authors reviewed and approved the final manuscript.

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Conflict of Interest

The authors declare no conflict of interest.

AI Declaration

Generative artificial intelligence tools were used solely to assist with language editing, grammatical refinement, and improvement of the clarity and readability of the manuscript. AI tools were not used to generate research data, conduct data analysis, or make substantive interpretations of the findings. The authors reviewed and verified the final manuscript and take full responsibility for its content.

Data Availability

The data supporting the findings of this study consist primarily of students' learning-obstacle diagnostic test responses, classroom observation records, interview transcripts, and related instructional documents. Due to ethical considerations and the need to protect the privacy and confidentiality of the student participants, the complete dataset is not publicly available. De-identified data may be made available by the corresponding author upon reasonable request, subject to ethical and institutional considerations.

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