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Original Article

The Impact of augmented reality on the learning of polyhedral: An approach based on didactical engineering and instrumental genesis

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Abstract
Despite the relevance of spatial skills in mathematics education, upper-secondary students face persistent difficulties, especially in the manipulation and conceptual understanding of three-dimensional objects. This study aims to design, implement, and analyze a didactic sequence mediated by Augmented Reality (AR) for the learning of polyhedral. The novelty lies in the analysis of the underlying cognitive processes through the framework of Instrumental Genesis, basing the design on the principles of Didactic Engineering. The research adopts a mixed-method and quasi-experimental approach. A pre-test and post-test were administered to a sample of fourth-semester upper-secondary students (n=12), complemented by an exhaustive qualitative analysis of the interaction with the GeoGebra 3D AR tool. Pre-test findings confirmed student weaknesses, showing only 25% success on measurement and dimensioning tasks. The post-intervention analysis demonstrated a significant and positive impact of the didactic sequence, evidenced by the total adaptation and instrumentalization of the AR tool. This resulted in a favorable evolution of cognitive schemes and a noticeable improvement in spatial visualization skills. The findings suggest that successful technological integration in 3D geometry must be guided by rigorous theoretical design and a detailed analysis of knowledge construction mediated by the instrument, providing empirical evidence for the implementation of AR in the mathematics classroom.

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1. Introduction

The development of spatial skills constitutes a fundamental component of mathematics learning that has garnered considerable attention in contemporary education research. For several decades, students have encountered significant difficulties in developing spatial abilities as part of their learning challenges, which consequently impacts their overall conceptual understanding of geometry (García & López, 2008). This phenomenon reflects the inherent complexity of the geometric learning process, necessitating more comprehensive pedagogical approaches that are adaptive to students' cognitive development and responsive to the evolving demands of mathematical literacy in the digital age.

In contemporary geometry teaching practices, traditional approaches remain predominantly focused on metric aspects such as perimeter, area, and volume, with excessive emphasis on procedural calculations (García & López, 2008). This conventional methodology creates substantial barriers to students' conceptual understanding formation by limiting their holistic grasp of geometric ideas and spatial relationships. Consequently, students tend to learn mechanical calculations and derivations without developing deep comprehension of the underlying geometric principles. This condition not only hinders the development of higher-order thinking skills but also fails to cultivate intrinsic motivation during the learning process (Su et al., 2022). The resulting gap between procedural knowledge and conceptual understanding represents a critical challenge in mathematics education that demands innovative pedagogical interventions.

The role of visual representations in geometry learning has long been recognized as a crucial element in the construction of mathematical concepts. In traditional approaches to geometry teaching and learning, images and visual representations play an essential role in the construction of geometric concepts (Vakaliuk et al., 2020). However, the limitations of static representations in conventional media often fail to accommodate the complexity of dynamic and multidimensional spatial concepts, particularly when dealing with three-dimensional objects such as polyhedra. These limitations underscore the urgency for integrating more innovative pedagogical approaches that are responsive to twenty-first-century learning needs and capable of providing interactive, dynamic visualizations that facilitate deeper spatial reasoning.

The advancement of digital technologies has opened new paradigms in mathematics pedagogy, particularly in the domain of geometry education. The integration of technological tools into geometry teaching-learning methodologies has emerged as a topic of growing interest in mathematics education research (Vakaliuk et al., 2020; Su et al., 2022). Among these technological innovations, Augmented Reality (AR) has demonstrated significant potential to transform geometry learning by enabling students to manipulate and visualize three-dimensional geometric objects in real-time, thereby bridging the gap between abstract concepts and concrete experiences (Sudirman et al., 2025; 2024). Various empirical studies have focused on comparing the use of technological resources with traditional methods, including comparisons with conventional textbooks (Su et al., 2022) and situated problem-solving approaches (Cangas et al., 2019). However, gaps remain in the comprehensive understanding of how technology, specifically AR, can be optimally integrated within structured pedagogical frameworks that address both theoretical and practical dimensions of learning.

The effectiveness of learning enhancement through systematic planning depends on teachers' ability to integrate appropriate stages, techniques, and instruments throughout the assessment process. Ayala and Portillo (2012) identified six essential components in designing comprehensive learning plans: first, understanding of competencies and pedagogical approaches; second, establishment of expected learning outcomes; third, content articulated in the curriculum; fourth, work methodology to be implemented; fifth, instructional resources and materials to support activity development; and sixth, the most appropriate assessment techniques and instruments (Ayala & Portillo, 2012). This comprehensive framework provides a foundation for systematically integrating innovative technologies into pedagogical practice while maintaining rigorous educational standards.

Despite significant advances in research on technology integration in geometry learning, there remains a critical need for deeper investigation into the practical implementation and effectiveness of AR-based approaches within structured learning contexts. Furthermore, theoretical frameworks that can guide the design and implementation of AR interventions are essential for ensuring pedagogical coherence and learning effectiveness. This study aims to explore and analyze the impact of Augmented Reality on polyhedral learning through the lens of Didactical Engineering and Instrumental Genesis theories, examining how these theoretical frameworks can inform the design of AR-enhanced learning experiences and investigating their implications for students' spatial skill development and conceptual understanding of three-dimensional geometric objects.

2. Theoretical Framewok

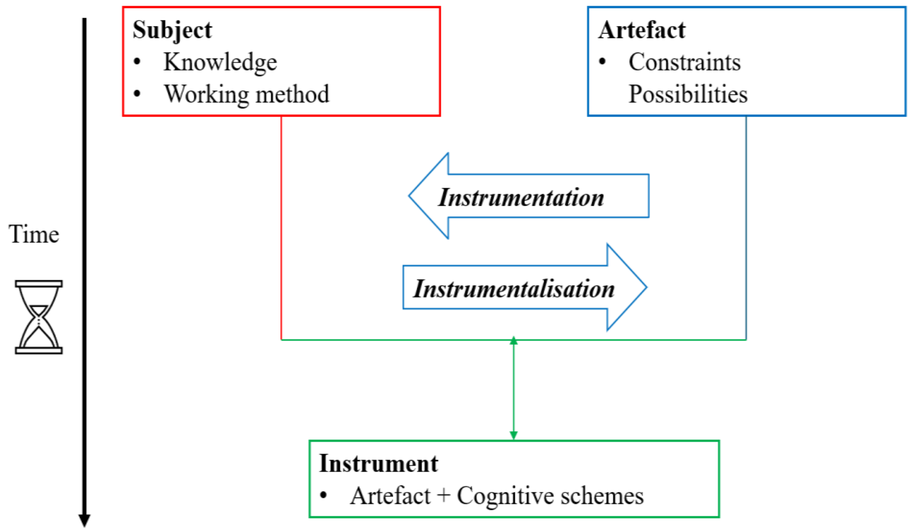
In 1997, Ronald Azuma, as cited in Gómez-vargas et al. (2018), established three fundamental characteristics that define AR technology: the seamless combination of reality and virtuality, real-time interactivity with the user, and the precise registration and rendering of three-dimensional objects. These characteristics distinguish AR from other technological approaches and establish the foundational requirements for effective AR implementation in educational settings. Building upon these foundational characteristics, Gómez-vargas et al. (2018) identified functional considerations that any effective AR application should achieve: accurately determining the current state of both the physical environment and virtual objects, enabling visualization in which virtual and real elements are coherently combined, and providing users with the convincing impression that virtual elements are integral parts of the physical environment.

Empirical evidence supports the educational efficacy of AR across various disciplines and learning contexts. The SMART application, implemented in 2008 by Portuguese researchers, presented real-time representations of transportation vehicles and animals, producing positive impacts on students with low academic performance (Gómez-vargas et al., 2018). In the field of astronomy, AR-enabled mobile devices have been successfully employed as learning tools for exploring the solar system (Baez & González, 2016). A pilot study comparing the visualization of three-dimensional objects through traditional methods versus AR demonstrated significantly improved learner performance through interactive engagement with virtual objects, highlighting AR's potential to enhance spatial reasoning and conceptual understanding. These findings suggest that AR technologies can address longstanding challenges in teaching abstract and spatial concepts by making them more accessible and manipulable for learners.

The incorporation of any technological instrument into didactic activities requires a systematic methodology that enables learners to appropriate it effectively. The Instrumental Genesis framework, developed within the broader context of instrumental approach theory, provides a structured conceptual foundation for understanding how learners transform technological artifacts into functional cognitive instruments. This theoretical perspective is particularly relevant for AR integration in mathematics education, as it addresses the complex relationship between technological tools and the cognitive schemes learners develop through their use.

According to Artigue (2002), an artifact possesses no inherent instrumental value for an individual; rather, it becomes an instrument through a developmental process known as instrumental genesis. This process involves the construction of personal schemes and the appropriation of pre-existing social schemes associated with the artifact's use. The concept of an instrument fundamentally differs from the material or symbolic object upon which it is based, the latter being the artifact. An instrument is thus a hybrid entity, composed partly of the physical artifact and partly of the cognitive processes and mental schemes involved in carrying out specific tasks (Artigue, 2020; Guin & Trouche, 2002). This distinction is crucial for understanding how students interact with AR technology, as it emphasizes that the educational value lies not in the technology itself but in the cognitive structures students develop through purposeful engagement with it.

Figure 1
Modelling of Instrumental Genesis. Source: Author's own elaboration



This theoretical perspective reveals that, consistent with Artigue's foundational principle, an artifact such as an AR application acquires significance only when there is a genuine need or purpose for its use within a learning context. During the utilization of such an artifact in specific situations or tasks, learners construct personal schemes, such as algorithms to determine its operation, application, and use, or appropriate already existing schemes by observing and internalizing how the tool is employed by others or demonstrated by teachers. This construction process is neither immediate nor automatic; it requires carefully designed didactic situations that promote meaningful interaction with the technological tool.

Instrumental Genesis operates bidirectionally during the creation of schemas for artifact utilization (Artigue, 2020; Guin & Trouche, 2002; Aguilar et al., 2017). The first process, instrumentalization, is directed toward the artifact itself, whereby users discover its potential affordances and may transform or adapt it for specific uses. Through instrumentalization, learners explore the artifact's features, customize its settings, and develop personalized strategies for its application (Artigue, 2002). In the context of AR for polyhedral learning, instrumentalization might involve students discovering how to rotate, scale, or manipulate virtual polyhedra, adjusting viewing angles, or exploring different visualization modes available in the application.

The second process, instrumentation, is directed toward the subject, leading to the development or appropriation of instrumented action schemes that progressively consolidate into techniques. These techniques enable effective responses to specific tasks and problem-solving situations (Guin & Trouche, 2002). In polyhedral learning with AR, instrumentation involves the development of cognitive schemes for using AR visualizations to understand geometric properties, identify relationships between faces and vertices, or solve spatial problems. The interplay between instrumentalization and instrumentation forms a dialectical process through which technological artifacts become genuine cognitive instruments that mediate mathematical thinking and learning.

Instrumental Genesis is considered a highly complex theoretical framework due to the non-observable nature of the cognitive schemas constructed by individuals when transforming artifacts into instruments. These internal mental structures cannot be directly observed, necessitating the use of complementary methodological approaches, such as task-based interviews, observational protocols, and analysis of students' interactions with the technology, to reveal and analyze the schemes being developed. This methodological challenge requires researchers to design studies that provide windows into students' cognitive processes, making visible the otherwise invisible transformations occurring during instrumental genesis.

For this study, GeoGebra served as the primary technological platform for implementing AR-enhanced polyhedral learning experiences. GeoGebra is free mathematical software created by Markus Hohenwarter and has been available since 2001 (Murcia, 2012). It has become one of the most widely adopted programs in mathematics teaching and learning worldwide, owing to its versatility, accessibility, and continuous development. The software's widespread adoption reflects both its technical capabilities and its alignment with contemporary pedagogical approaches that emphasize dynamic, interactive, and multiple representational approaches to mathematical learning.

GeoGebra offers three interconnected perspectives for each mathematical object: a graphical view, a numerical-algebraic view, and a spreadsheet view, enabling multiple representations that support conceptual understanding (Arteaga-Valdés et al., 2019; Sugiarni et al., 2025). This multi-representational approach aligns with research on mathematical cognition suggesting that deep understanding emerges from the ability to flexibly move between different representational systems. González et al. (2017) assert that GeoGebra contributes substantially to improving teaching and learning methodologies, as well as the resolution of mathematical problems, by providing valuable information through dynamic graphical representations. The dynamic nature of GeoGebra representations allows students to explore mathematical relationships through manipulation and observation, fostering an investigative approach to learning.

The integration of AR functionality within GeoGebra has been extensively explored in the teaching of algebra, geometry, and calculus (Muñoz Casado, n.d.; Muñoz, 2019). For this investigation, GeoGebra 3D was primarily employed due to its integrated AR capabilities, which enable students to

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visualize and manipulate three-dimensional polyhedra in augmented environments. This functionality is particularly valuable for polyhedral learning, as it addresses the well-documented challenge students face when transitioning from two-dimensional representations to spatial understanding of three-dimensional objects. By allowing students to view polyhedra from multiple angles, manipulate them in space, and observe their properties dynamically, GeoGebra AR provides affordances that traditional static representations cannot offer.

The synthesis of Didactical Engineering and Instrumental Genesis provides a comprehensive framework for this study. Didactical Engineering offers a methodological structure for designing, implementing, and analyzing didactic sequences, ensuring that AR integration is purposeful and pedagogically grounded. Instrumental Genesis, meanwhile, provides theoretical insight into how students appropriate AR technology as a cognitive instrument for learning polyhedra, focusing attention on the cognitive transformations that occur during technology-mediated learning. Together, these frameworks enable a systematic examination of both the pedagogical design and the cognitive processes involved in AR-enhanced geometric learning, ensuring that technological integration serves genuine educational purposes rather than merely adding technological novelty to traditional instruction.

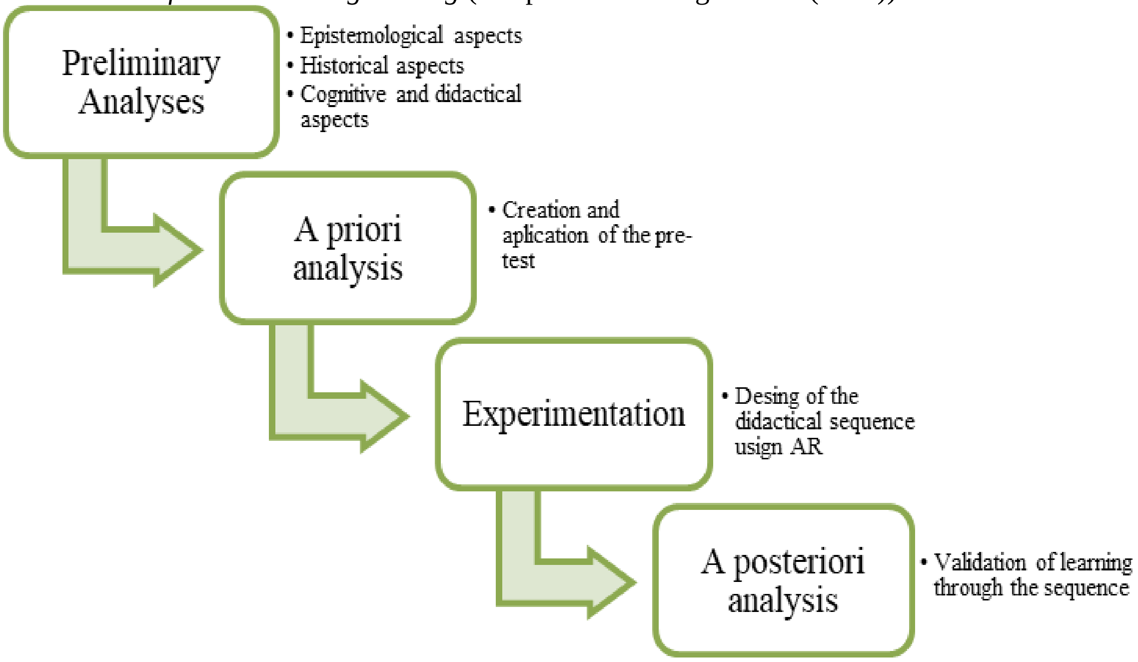
3. Method

3.1 Research Design

This study employs a mixed-methods approach with a dominant qualitative component embedded within a quasi-experimental design. The quasi-experimental component, involving pre-test and post-test comparisons, was utilized to quantitatively assess the impact of the AR-enhanced didactic sequence on students' geometric reasoning skills, particularly in the context of polyhedral learning. Simultaneously, the dominant qualitative component was essential for analyzing the cognitive processes and interaction dynamics during the intervention, specifically focusing on the processes of Instrumental Genesis. This methodological approach allows for not only claiming effectiveness in terms of learning outcomes but also for deeply understanding the modification and development of students' mental schemes during instrumentalization and instrumentation when using the GeoGebra 3D AR tool.

The methodological framework of this study is grounded in Didactical Engineering, which provides a systematic structure for designing, implementing, and analyzing educational interventions. This theory establishes four interconnected phases (Artigue et al., 1995): Preliminary Analysis, A Priori Analysis, Experimentation, and A Posteriori Analysis, as illustrated in Figure 2. These phases guide the entire research process, from initial conceptualization through final evaluation, ensuring pedagogical coherence and theoretical rigor throughout the investigation.

Figure 2 *Phases of Didactical Engineering* (Adapted from Artigue et al. (1995))



The Preliminary Analysis phase involved an examination of three critical dimensions that provided the foundations for the design of the didactic sequence. The epistemological dimension addressed the conceptualization of the properties and classification of polyhedra, examining the mathematical structure and historical development of these concepts. The cognitive dimension investigated the documented difficulties students encounter in learning about polyhedra, drawing on existing literature regarding spatial reasoning challenges. The didactical dimension explored pedagogical approaches to teaching polyhedra using Augmented Reality, considering both the affordances and constraints of AR technology in geometric education.

During the A Priori Analysis phase, a pre-test was designed to evaluate students' preliminary knowledge and establish a baseline for the intervention. This diagnostic instrument served as the foundation for developing the didactic sequence and identifying specific areas requiring pedagogical attention. The pre-test consisted of six items intended to assess geometric reasoning in three-dimensional space at different cognitive levels. The evaluation framework was based on the four types of reasoning established by Pittalis and Christou (2010), organized into four sections, each corresponding to a specific type of reasoning: identification and construction of nets, manipulation of polyhedra, knowledge of three-dimensional polyhedral properties, and measurement and dimensioning of polyhedra.

The pre-test was adapted from the validated instrument developed by Pittalis and Christou (2010) and modified to fit a 40-minute session with a fourth-semester upper secondary school group. The instrument was structured as follows: Items 1 and 2 (R1 and R2) corresponded to the analysis of the development of figures from two-dimensional to three-dimensional representations; Items 3 and 4 (R3 and R4) focused on the mental manipulation of three-dimensional objects; Item 5 (R5), which included three regular polyhedra, evaluated knowledge of polyhedral properties; and Item 6 (R6), composed of four sub-items, assessed measurement and magnitude in three-dimensional objects. The complete test was administered in a 50-minute group session to ensure students had adequate time for thoughtful responses.

The Experimentation phase involved the implementation of the AR-enhanced didactic sequence based on the results obtained from the pre-test. A comprehensive didactic sequence supported by a structured workbook was developed for the integration of Augmented Reality, with the primary aim of enhancing student learning of polyhedra through interactive, dynamic visualization and manipulation of three-dimensional geometric objects. The didactic sequence was designed to progressively facilitate students' instrumental genesis, moving from initial exploration of the AR tool to sophisticated use of it as a cognitive instrument for geometric reasoning.

The A Posteriori Analysis phase focused on the evaluation of the didactic sequence within the classroom setting, assessing both the instrumentation and instrumentalization processes of Instrumental Genesis through the development of mental schemes. During this stage, students were asked to document each of the activities they carried out, with the aim of analyzing their individual performance, collaborative interactions during tasks, and the evolution of their use of the AR tool. This documentation provided crucial data for understanding how students transformed the GeoGebra AR artifact into a functional instrument for geometric learning.

2.2 Participants

The study was conducted with students from a fourth-semester upper secondary education class in the state of Puebla, Mexico. The initial experimental group consisted of 25 students who participated in the complete didactic sequence. However, the final analytical sample for in-depth qualitative analysis was composed of 12 students who met stringent selection criteria. This reduction in sample size reflects the methodological requirements for conducting rigorous qualitative analysis of Instrumental Genesis processes, which demands complete and consistent data across all phases of the intervention.

The selection criteria for the final analytical sample were established to ensure the validity and reliability of the qualitative data related to Instrumental Genesis. Three primary criteria were applied: first, consistent attendance at all sessions of the didactic sequence to ensure continuity of the instrumental genesis process; second, complete submission of all required activities, including worksheets, AR interaction logs, and reflection exercises; and third, demonstrable, prolonged interaction with the GeoGebra 3D AR tool during the tasks, as evidenced through observation records and digital activity logs. Students who did not meet all three criteria were excluded from the qualitative analysis to maintain the integrity of the instrumental genesis investigation, though their quantitative data from pre-test and post-test assessments were retained for comparative purposes.

The participating group was considered relatively homogeneous in terms of academic background, as all students were enrolled in the same educational level and had completed the same prerequisite mathematics courses. This homogeneity helped control for potential confounding variables related to prior mathematical knowledge and experience. Prior to the intervention, informed consent was obtained from both students and their legal guardians, following institutional ethical protocols for educational research. Students were informed that their participation was voluntary and that they could withdraw from the study at any time without academic penalty.

2.3 Data Collection

Data collection employed multiple instruments and methods to capture both quantitative learning outcomes and qualitative processes of Instrumental Genesis. This triangulation of data sources strengthened the validity of findings and enabled comprehensive analysis of both the effectiveness and the mechanisms of the AR intervention.

The quantitative data was collected using a validated test designed to measure geometric reasoning in three-dimensional space, which was adapted from the work of Pittalis and Christou (2010). This instrument assesses six key components of spatial reasoning, including visualization of three-dimensional objects, mental manipulation and rotation, understanding of geometric properties, identification and construction of nets, measurement capabilities, and dimensioning of polyhedra. The instrument was administered as a pre-test prior to the intervention to establish baseline geometric reasoning abilities and was planned to be administered as a post-test following the completion of the didactic sequence to measure gains in spatial reasoning skills. Both administrations were conducted under standardized conditions to ensure comparability of results.

The qualitative data collection involved multiple complementary approaches designed to capture different aspects of the Instrumental Genesis process. First, detailed field notes and structured observation records were maintained throughout all sessions to document student interactions with the AR tool, patterns of collaboration, problem-solving strategies, and verbal expressions of geometric reasoning. These observations were guided by a protocol focused on identifying evidence of instrumentalization, such as students discovering and adapting features of the GeoGebra AR tool, and instrumentation, such as the development of systematic techniques for using AR to solve geometric problems.

Second, video recordings of group discussions and individual student work sessions were systematically collected to enable retrospective analysis of the evolution of communication schemes and collaborative learning processes. These recordings captured both verbal discourse and physical interactions with the AR technology, providing rich data on how students articulated their geometric thinking and negotiated meaning with peers. The video data also allowed for microgenetic analysis of critical moments in the instrumental genesis process, such as breakthroughs in understanding or the emergence of new utilization schemes.

Third, systematic collection and analysis of the digital artifacts created by students using the AR tool, specifically the GeoGebra 3D files they produced during various tasks, served as material evidence of their growing instrumental mastery. These artifacts were analyzed for sophistication of construction, accuracy of geometric representations, and creative applications of AR functionality. Additionally, students' written responses in their workbooks, including explanations of their problem-solving processes and reflections on their use of the AR tool, provided complementary data on their developing understanding.

Finally, semi-structured interviews were conducted with selected students following the completion of the didactic sequence to gain deeper insight into their experiences with the AR tool and their perceptions of how it influenced their learning. These interviews explored students' awareness of their own instrumental genesis, their strategies for using AR to solve geometric problems, and their assessments of the advantages and limitations of AR for learning about polyhedra.

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2.4 Data Analysis

Data analysis proceeded through both quantitative and qualitative pathways, with findings from each approach informing and enriching the other in an iterative process of interpretation. The quantitative analysis focused on measuring changes in geometric reasoning performance, while the qualitative analysis explored the processes and mechanisms underlying these changes.

For the quantitative component, pre-test and post-test scores were analyzed using descriptive statistics to characterize the overall distribution of geometric reasoning abilities before and after the intervention. Paired sample t-tests or non-parametric equivalents were employed to determine whether statistically significant improvements occurred in overall geometric reasoning and in specific subscales corresponding to the four types of reasoning identified by Pittalis and Christou (2010). Effect sizes were calculated to assess the practical significance of any observed improvements. Analysis was conducted both for the full experimental group and for the analytical subsample to examine whether patterns differed between these groups.

The qualitative analysis was conducted through a multi-layered approach grounded in the theoretical framework of Instrumental Genesis. The primary analytical focus was on identifying and characterizing the instrumentalization and instrumentation processes as students transformed the GeoGebra AR artifact into a cognitive instrument for geometric learning. Video recordings and field notes were initially reviewed to identify critical episodes demonstrating evidence of instrumental genesis, such as moments when students discovered new functionalities, adapted the tool for specific purposes, or developed systematic techniques for its use.

Detailed coding of these critical episodes was conducted using a framework derived from Instrumental Genesis theory, with codes distinguishing between instrumentalization schemes (artifact-directed processes) and instrumentation schemes (subject-directed processes). For instrumentalization, codes captured students' explorations of AR features, customizations of settings, development of personal strategies for manipulation, and adaptations of the tool for specific tasks. For instrumentation, codes identified the emergence of systematic techniques, the development of mental schemes for using AR to support geometric reasoning, and the progressive refinement of instrumented action schemes over time.

Students' digital artifacts (GeoGebra 3D files) were analyzed using qualitative content analysis to assess the sophistication and accuracy of their geometric constructions, as well as evidence of creative or advanced applications of AR functionality. These artifacts were examined chronologically for each student to trace the evolution of their instrumental mastery across the didactic sequence. Written responses in workbooks were analyzed using thematic analysis to identify patterns in students' explanations of geometric concepts and their reflections on the role of AR in their learning process.

Interview transcripts were coded using both deductive codes derived from Instrumental Genesis theory and inductive codes emerging from the data itself. This analysis sought to understand students' metacognitive awareness of their instrumental genesis, their explicit strategies for using AR as a learning tool, and their subjective assessments of the AR intervention's value. Cross-case analysis compared patterns across students to identify common trajectories of instrumental genesis as well as individual variations in the appropriation process.

Triangulation of data sources was employed to enhance the credibility and trustworthiness of findings. Convergence of evidence from observations, artifacts, interviews, and quantitative assessments strengthened conclusions about both the effectiveness of the intervention and the nature of the instrumental genesis processes. Discrepancies between data sources were examined as potential indicators of complexity or contextual variation in the appropriation process. Member checking was conducted by sharing preliminary interpretations with selected students to verify the accuracy of our understanding of their experiences and thought processes.

4. Results and Discussion

4.1 Results

The following section presents the results obtained from the pre-test, which reveal the deficiencies shown by students in geometric reasoning across different levels. The table displays the percentage of achievement demonstrated by students at each assessed level during the pre-test.

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Table 1
Levels assessed through the pre-test

Level	Goal	Item
Identification and construction of nets.	To identify, from a net, the polyhedron that is formed when the planes are assembled. To identify the unfolding of a polyhedron on a two-dimensional plane.	R1. To identify the development of a figure from 3D to 2D. Percentage achieved by the population: 100%. R2. To identify the 3D figure from its 2D development. Percentage achieved by the population: 100%.
Mental manipulation of polyhedra.	To mentally manipulate the polyhedron in order to identify the top, front, and side views.	R3. To mentally construct the identity of a figure from its top, front, and side views. Percentage achieved by the population: 78%. R4. To identify the side, top, and front views of a 3D figure. Percentage achieved by the population: 63%.
Knowledge of properties.	To recognize the properties of polyhedral, such as the number of faces, vertices, and edges.	R5. To identify the elements that make up regular polyhedral (faces, edges, and vertices). Percentage achieved by the population: 78–22%.
Measurement and dimensioning of polyhedral.	To calculate the areas and volumes of polyhedral, not only by using formulas but also through mental construction.	R6. To calculate the volume of two prisms and the space occupied by one figure within another. Percentage achieved by the population: 25%.

As shown in Table 1, it is observed that in items R1 and R2 (Figure 3 and Figure 4), which correspond to the level of identification and construction of nets, 100% of the students achieved satisfactory results; that is, all of them were able to construct the geometric nets from 2D to 3D and vice versa.

Figure 2
Item I: i) 3D polyhedron, ii) 2D structure, and iii) percentage accuracy chart.

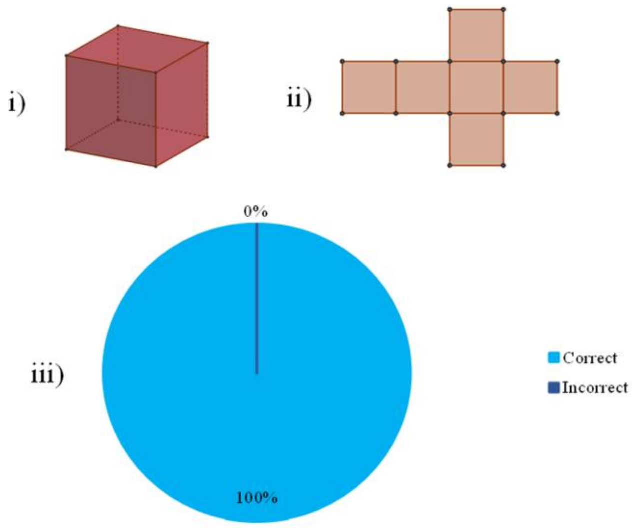

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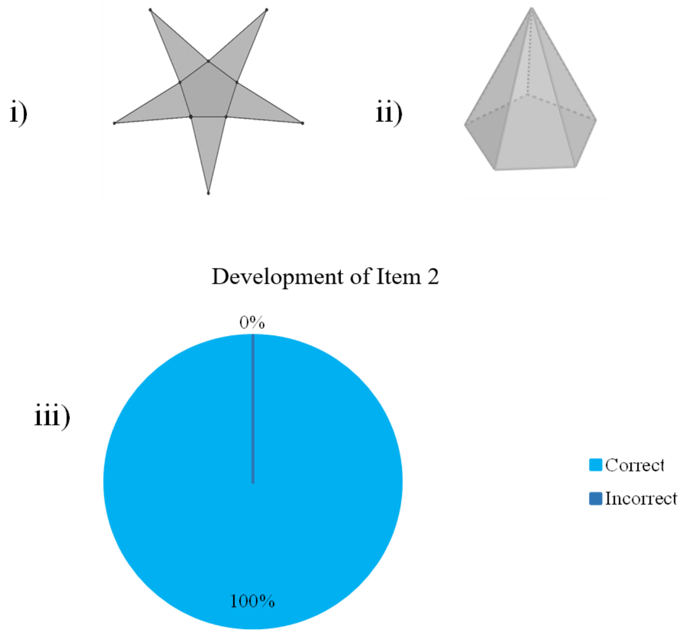
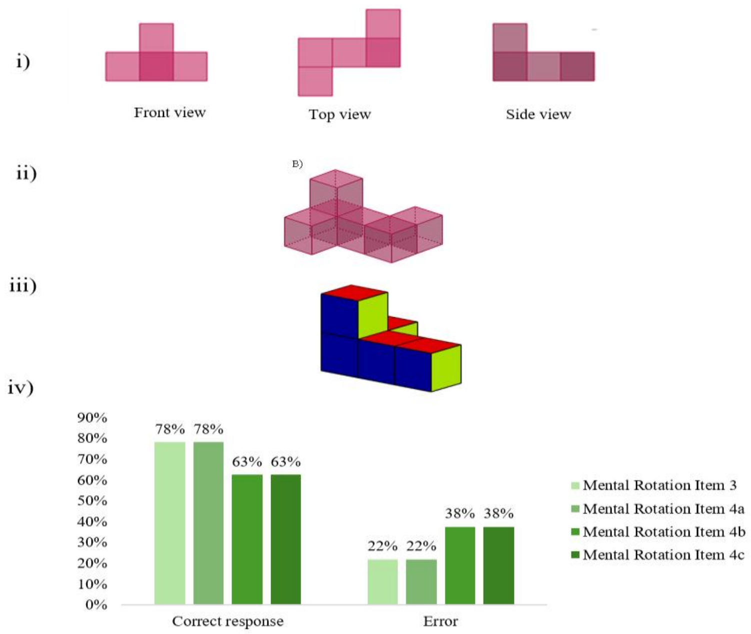
Figure 3
Item 2: i) 2D structure, ii) 3D polyhedron, and iii) percentage of correct answers chart



In items R3 and R4 (Figure 5), corresponding to the level of manipulation of 3D figures, there was a decrease in the percentage of correct responses, with 78% and 63% respectively for each item (see Table 1). In the first observation, the task of superimposing 2D images (top, side, and front views) proved difficult for the students; likewise, manipulating a 3D figure and constructing its two-dimensional views also presented challenges for them.
Figure 5
Items 3 and 4: i) Front, top, and side views, ii) 3D construction, and iii) 3D structure



In item R5 (Figure 6), which corresponds to the level of knowledge of properties, it was identified that, with regard to the three figures presented at this level, the student was unable to identify the number

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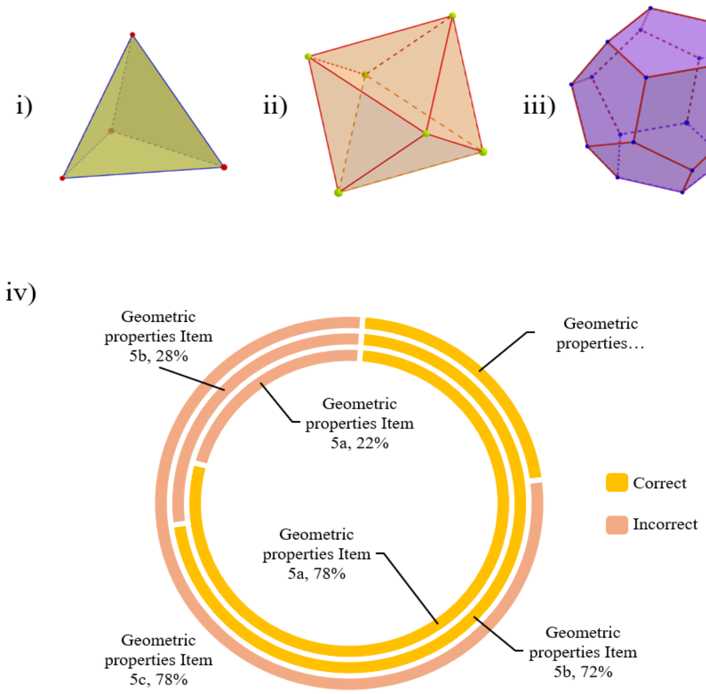
of faces, edges, and vertices in more complex figures, such as the icosahedron. However, for figures involving lower levels of mental manipulation difficulty, the process of counting the properties of regular polyhedra proved much easier (78%). For item R6, corresponding to the level of measurement and dimensioning of 3D figures (polyhedra), students achieved a 22% success rate, revealing confusion between area and volume. Based on these results, it was established that the main difficulties to be addressed through the didactic sequence, as well as those identified in the a priori analysis, were: the mental manipulation of 3D objects, the construction of two-dimensional nets from a 3D figure, and the measurement and dimensioning of 3D objects.
Figure 6.
Item 5: i) Item 5a, ii) Item 5b, iii) Item 5c, and iv) accuracy chart corresponding to the item



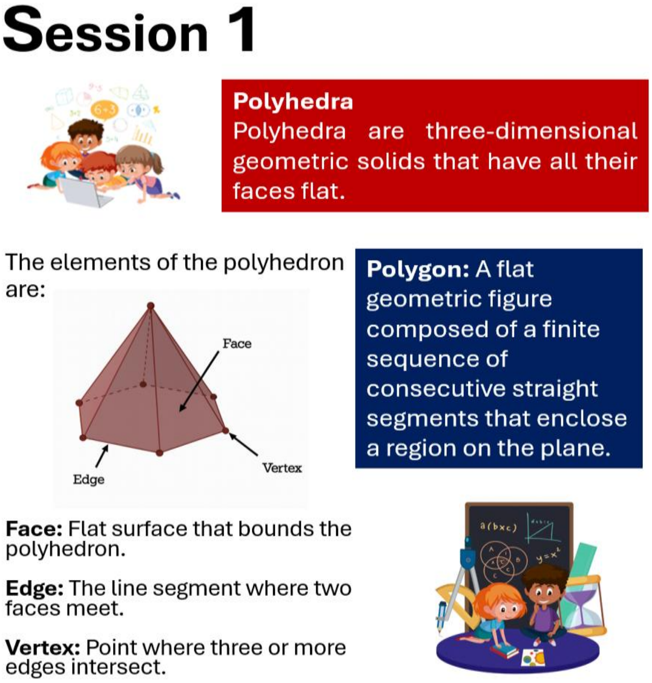
The results underpin the design of the didactic sequence, which begins with a high level of recognition of 2D figures but shows difficulties in the manipulation of 3D objects.

4.1.1 Design of the Didactic Sequence

A comprehensive didactic sequence was designed and structured into three progressive sessions, each incorporating the essential pedagogical phases of introduction, development, and closure to ensure coherent learning progression. The sequence systematically addressed the topic of polyhedra and their geometric properties, with particular attention to the difficulties identified through pre-test analysis, specifically those related to visualization and mental manipulation of three-dimensional objects. All activities were intentionally designed to incorporate Augmented Reality through GeoGebra 3D, with the dual aims of reducing identified cognitive obstacles and strengthening students' spatial reasoning and geometric skills through interactive, dynamic engagement with virtual polyhedra.

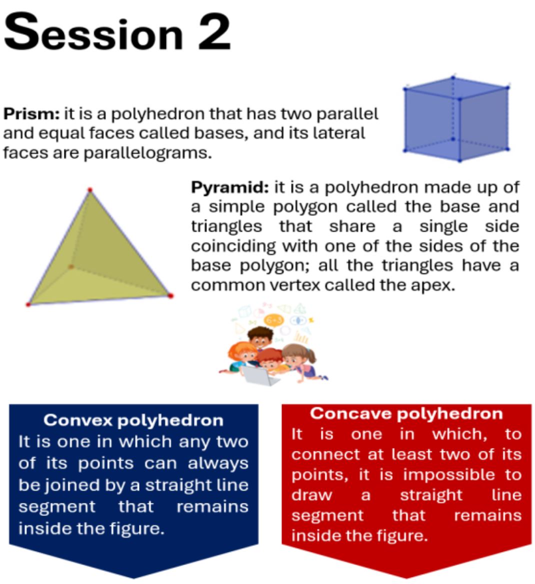
The first session focused on introducing students to the GeoGebra 3D interface and fundamental AR functionalities while simultaneously reviewing essential concepts related to polyhedra. This foundational session was strategically divided into three sequential components designed to progressively build technical competence and geometric understanding: construction of segments in three-dimensional space, construction of polygons as two-dimensional foundations, and culminating in the construction of complete polyhedra, thereby establishing both technological familiarity and conceptual groundwork for subsequent sessions.

Figure 7
Session 1 of the workbook



In the second session, students engaged with more sophisticated geometric concepts, specifically examining the structural properties and classifications of prisms and pyramids, while also exploring the fundamental distinction between concave and convex polyhedra through AR-enhanced visualization. This session was strategically divided into two complementary parts: the systematic construction of various types of prisms with different polygonal bases, and the guided construction of pyramids with varying heights and base configurations. Through hands-on manipulation of AR models, students developed deeper understanding of how these polyhedra differ in their geometric properties and spatial characteristics.

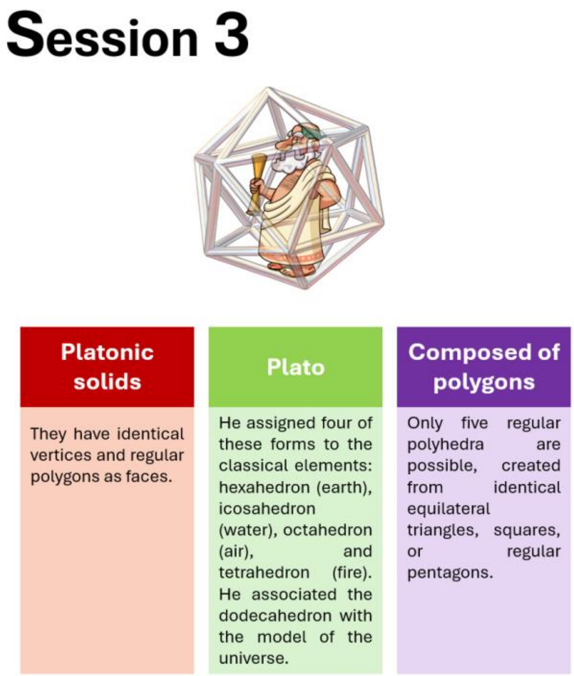
Figure 8
Session 2 of the workbook



In the third and culminating session, students explored the mathematically significant category of Platonic solids, investigating their unique properties of regularity, symmetry, and historical importance in geometry. This advanced session was methodically divided into sequential construction activities utilizing the AR tool to build three complex regular polyhedra: the tetrahedron with its four triangular faces, the dodecahedron comprising twelve pentagonal faces, and the icosahedron constructed from twenty triangular faces. Through these AR-enhanced constructions, students not only developed technical proficiency but also gained appreciation for the mathematical elegance and structural relationships inherent in these perfect geometric forms, thereby deepening their understanding of three-dimensional spatial reasoning and geometric regularity

Figure 9

Session 3 of the workbook. Source: Author's own elaboration



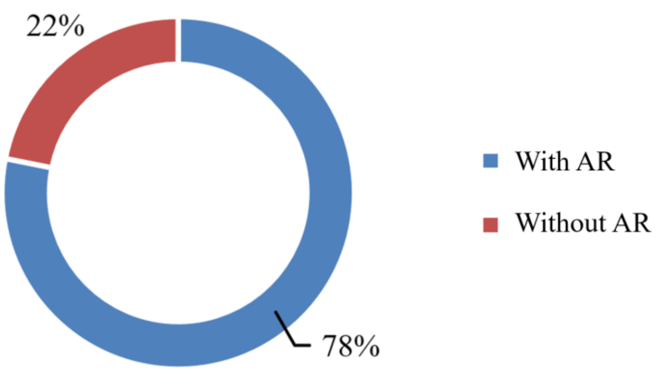
Each session was carefully structured to include diverse activities supported by Augmented Reality technology, strategically designed to facilitate students' exploration of GeoGebra 3D tools while simultaneously fostering the progressive development of essential geometric skills and spatial reasoning abilities. The activities were sequenced to gradually increase in complexity, moving from basic manipulation and visualization tasks to more sophisticated construction and analysis challenges. Every session incorporated a carefully designed didactic situation that served dual purposes: formatively assessing the learning achieved during the session and summatively evaluating the geometric knowledge students had acquired. These didactic situations were grounded in the principles of Didactical Engineering, providing authentic problem-solving contexts that required students to apply their developing instrumental mastery of the AR tool. The integration of assessment within each session enabled continuous monitoring of both learning outcomes and the progression of instrumental genesis processes, allowing for responsive adjustments to instruction when necessary and providing rich data on how students' utilization schemes evolved across the intervention.

The implementation of these sessions with the experimental group incorporated innovative documentation protocols designed to capture evidence of cognitive processes and instrumental genesis. Each student was explicitly instructed to activate screen recording functionality on their mobile devices throughout all AR-based activities, thereby creating detailed digital records of their interactions with the GeoGebra 3D tool, including navigation patterns, manipulation strategies, construction sequences, and problem-solving approaches. This screen recording methodology provided unprecedented access to students' instrumental behaviors and decision-making processes as they engaged with geometric tasks in augmented reality environments. Following completion of each session, students were required to upload their screen recordings to a designated shared repository, creating a comprehensive video database that facilitated systematic organization, retrieval, and detailed qualitative analysis. These video records became primary data sources for analyzing instrumentalization processes, such as tool exploration and feature discovery, as well as instrumentation processes, including the development and refinement of systematic techniques for using AR to support geometric reasoning and problem-solving.

4.1.2 Analysis of Audiovisual Material

The analysis of audiovisual material constituted a rigorous and systematic process centered on constructing detailed schemes and developing theoretically grounded interpretations of the activities carried out by students during their engagement with the AR-enhanced didactic sequence. This analytical process was essential for documenting evidence of instrumental genesis and understanding how students transformed the GeoGebra 3D artifact into a functional cognitive instrument for geometric learning. The selection of videos for in-depth qualitative analysis was based on three stringent inclusion criteria designed to ensure data quality and analytical validity. First, the recorded work must have been conducted using the Augmented Reality functionality of GeoGebra 3D, demonstrating authentic engagement with the target technology rather than alternative features of the software. Second, students must have attended all sessions in which the AR tool was employed, ensuring continuity in their instrumental genesis trajectory and completeness of their developmental process across the entire intervention. Third, students must have submitted all required activities and didactic situations punctually and in the specified format, indicating commitment to the learning process and providing complete documentation for analysis. Based on these rigorous selection criteria, a specific subset of students from the original experimental group was identified for comprehensive qualitative analysis, ensuring that analytical conclusions regarding instrumental genesis were grounded in high-quality, complete, and representative data sets.

Figure 10
Number of students with the AR tool on their mobile device



Regarding the first criterion established for the evaluation and selection of students for comprehensive analysis, empirical observation revealed that 78% of the participants were successfully able to utilize Augmented Reality functionality on their personal mobile devices during the intervention sessions. However, a significant technological limitation emerged that constrained full participation, as not all students' devices proved compatible with the GeoGebra 3D AR tool due to varying hardware specifications, insufficient processing capabilities, outdated operating system versions, or lack of required sensors such as gyroscopes and accelerometers essential for AR functionality. This technological barrier, while representing a practical constraint of implementing AR in educational contexts with diverse student resources, also highlighted the importance of considering device compatibility and accessibility when designing technology-enhanced interventions. The incompatibility issues affected approximately 22% of students, preventing them from engaging fully with the AR-enhanced activities and consequently excluding them from the primary analytical sample, though alternative accommodations were provided to ensure their continued participation in the broader learning experience.

Following systematic application of all three selection criteria, it was determined that of the 25 students initially enrolled and expected to complete the activities using Augmented Reality, only 18 demonstrated consistent attendance at all scheduled sessions throughout the intervention period, and ultimately, merely 12 students successfully met all established criteria for inclusion in the rigorous qualitative research analysis focused on instrumental genesis processes. These 12 students formed the final analytical sample and were systematically classified according to researcher-developed criteria that emerged from preliminary data examination, enabling the identification of distinct cognitive schemes and utilization patterns that differed from initially expected developmental trajectories. This classification framework facilitated the detailed description of various learning schemes and instrumental genesis pathways, through which the complex development of geometric learning mediated by Augmented Reality could be comprehensively observed, documented, and theoretically interpreted. The diversity of schemes identified within this purposefully selected sample enriched the analysis by revealing multiple routes through which students appropriated the AR tool as a cognitive instrument for polyhedral learning.

4.1.3 Analysis of Didactic Situation 1

The first section of the workbook aimed to familiarize students with the GeoGebra artefact. The activities were carried out in the classroom, where the mobile device using the GeoGebra tool was projected, allowing students to observe while the teacher, supported by a selected student, demonstrated the resolution of the proposed exercises.

Subsequently, students were asked to individually complete a didactic situation, which stated:

> *"Using Augmented Reality, construct a polyhedron with ten faces and a volume of seventy-five cubic units. Remember to record your GeoGebra 3D process in order to evaluate your procedure."*

Through meticulous analysis of the construction process undertaken by Student 1, detailed observation revealed both the systematic manipulation of geometric objects and the specific cognitive obstacles encountered during task completion. The student's initial approach to constructing a three-dimensional polyhedron began with what appeared to be a logical two-step process: first, creating individual points positioned within the xy-plane coordinate system, and second, attempting to connect these discrete points using the polygon construction tool available in GeoGebra 3D. However, when executing the point-joining procedure, the student made a critical discovery that the resulting plane figure did not form a closed polygonal shape, as evidenced by gaps or discontinuities in the boundary (see Figure 11). This geometric error presented a fundamental obstacle because the extrusion function, necessary for transforming the two-dimensional polygon into a three-dimensional polyhedron, requires a properly closed planar figure as its foundation. Confronted with this impasse and recognizing the inability to proceed with extrusion, the student made the strategic decision to delete all previously created points and restart the construction process, demonstrating metacognitive awareness and adaptive problem-solving capacity.

During this reconstructive process, a significant moment of instrumental genesis occurred as the student gained deeper insight into the functional capabilities of the polygon tool itself. Specifically, the student realized that when utilizing the polygon construction tool directly, rather than pre-creating individual points, it became possible to correctly position vertices and simultaneously form the connecting edges in a single, integrated operation that automatically ensured closure of the geometric figure. This realization represented a crucial instrumentalization process, wherein the student discovered and appropriated a more efficient and effective functionality of the GeoGebra AR artifact. Consequently, the student adaptively modified their initial construction scheme, abandoning the two-step approach of creating points then connecting them, and instead adopting a streamlined methodology that relied exclusively on the integrated polygon tool for constructing planar figures. This adaptive transformation of technique exemplifies the development of instrumented action schemes central to instrumental genesis theory. The revised approach enabled the student to construct the polygon correctly with proper closure, subsequently allowing successful extrusion into a three-dimensional form and ultimate completion of the polyhedral construction task, thereby demonstrating how cognitive obstacles, when navigated through reflective practice, can catalyze instrumental development and conceptual learning.

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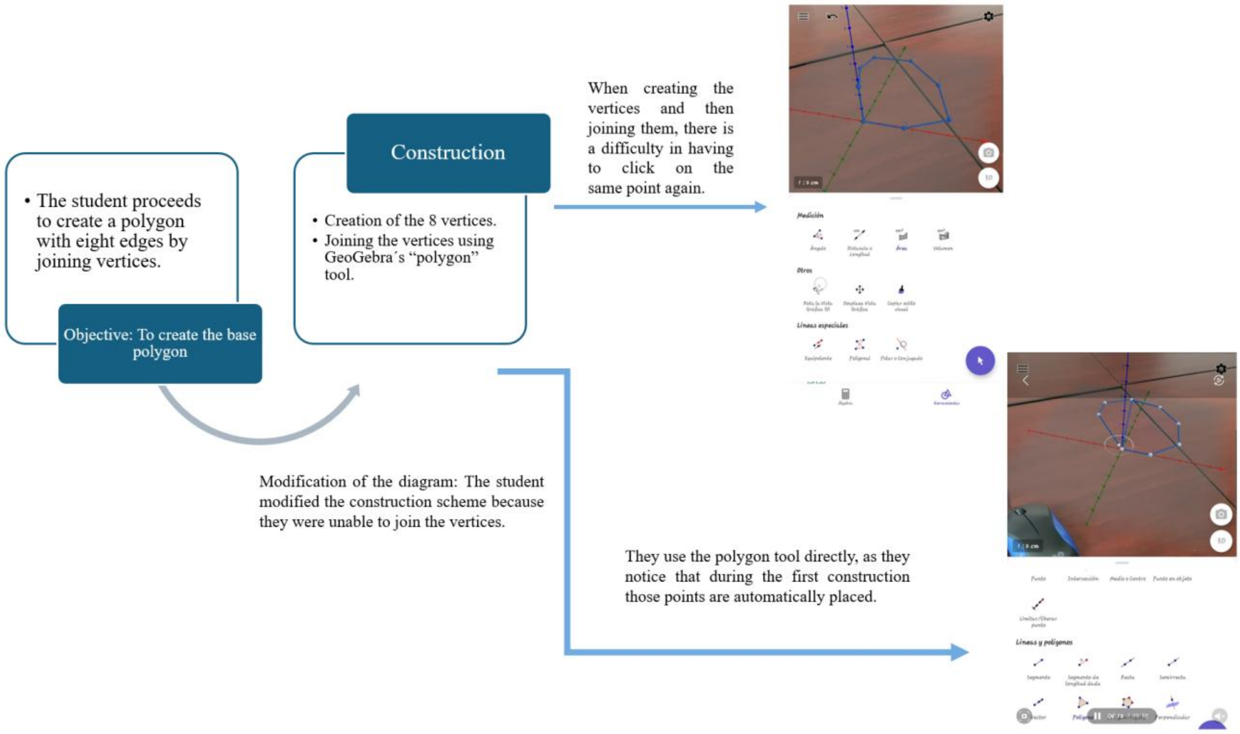
Figure 11
Mental scheme of the didactic situation from Session 1



Careful examination of Figure 12 reveals critical aspects of both the polyhedron construction process and the subsequent manipulation strategies employed to determine surface area calculations. The students demonstrated adaptive cognitive flexibility by fundamentally modifying their existing mental schemes when confronted with the demands of the task, illustrating a key aspect of instrumentation within the instrumental genesis framework. This scheme modification enabled them to successfully navigate the computational challenges inherent in calculating three-dimensional surface area using the AR tool. Through this adaptive process, students progressively developed more sophisticated instrumented action schemes that integrated geometric visualization, measurement techniques, and strategic manipulation of the virtual polyhedron, ultimately achieving successful task completion while simultaneously advancing their instrumental mastery of the GeoGebra 3D environment.

Another situation of interest in this research was that of Student 2, who performed the activity as follows:

> *During the task, the student displayed a different line of reasoning regarding the figure. Unlike their classmates, the student was unable to complete the established didactic situation. They began by drawing the vertices of the polygon, creating a decagon (polygon with ten sides). Throughout this process, the student attempted several times to connect the points, becoming frustrated when unable to join them and form their faces. The student assumed that the faces of a prism consisted only of the lateral faces, excluding the top and bottom bases. As a result, the student generated a polyhedron with twelve faces, without realizing this error.*

During the activity execution, the student focused exclusively on calculating the surface area and volume of the constructed polyhedron, deliberately assigning a height measurement of ten units to the three-dimensional object. However, upon encountering difficulties in obtaining immediate computational results through the AR tool, the student experienced frustration and cognitive impasse. Consequently, rather than persisting with alternative problem-solving strategies or seeking to modify their approach, the student prematurely abandoned the established construction scheme, ultimately

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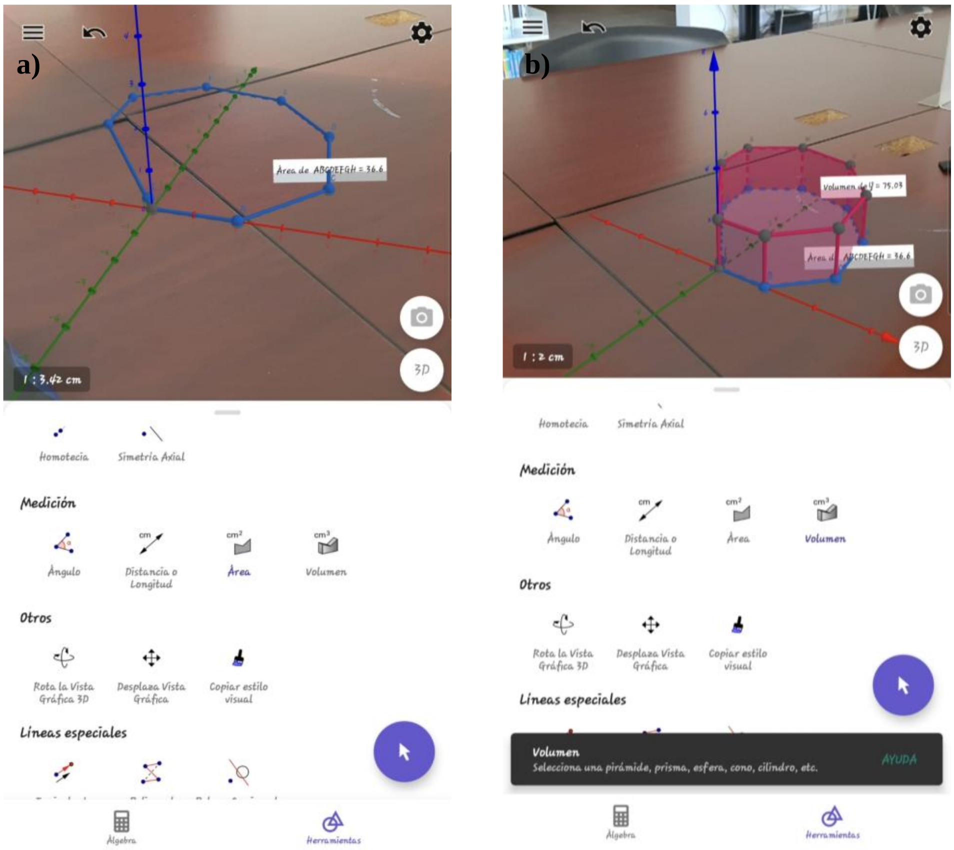
leaving the assigned task incomplete and unresolved, which suggests insufficient development of resilient instrumented action schemes necessary for navigating technical obstacles.

Figure 12

Student's result from the didactic situation in Session 1



4.1.4 Analysis of Didactic Situation 2

The primary objective of this activity was to develop students' ability to accurately identify different types of pyramids and to establish clear conceptual distinctions between concave and convex polyhedral structures through interactive AR-mediated exploration. During the instructional session, the entire group engaged in collaborative work under the teacher's guidance to construct various pyramidal forms using GeoGebra 3D's AR functionality. The pedagogical approach employed involved the teacher projecting exercises from Section 2 of the structured workbook onto a shared screen visible to all participants, while a deliberately selected student completed the construction tasks in real-time, serving as a live demonstration and reference model for the remainder of the group. This collaborative instructional strategy enabled peer observation of construction techniques, fostered discussion of geometric properties, and created opportunities for collective problem-solving when challenges arose. The approach exemplified social dimensions of instrumental genesis, wherein students appropriated utilization schemes not only through individual exploration but also through observing and internalizing techniques demonstrated by peers and the instructor.

The culminating didactic situation of Section 2 presented students with a sophisticated challenge that integrated multiple geometric concepts and required strategic application of AR tool affordances: "Construct a pyramid with a volume of 50 cubic units, whose concave base has an area of 5 square units." This task demanded understanding of volume relationships, precise base construction with specified area, and recognition of concavity properties. In addressing this situation, Student 3 demonstrated advanced instrumentalization by constructing a polygon with five vertices, strategically manipulating one vertex to create an interior angle of 180 degrees, thereby producing the required concave characteristic (see Figure 13). The student exhibited sophisticated control of the AR tool by

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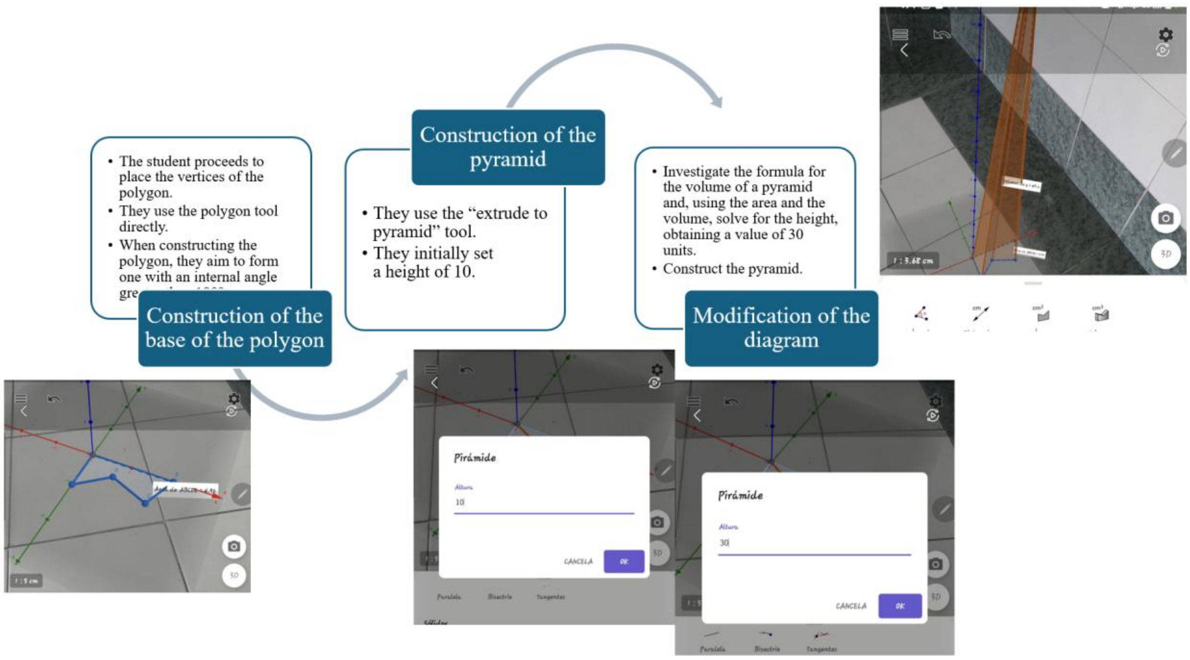
systematically manipulating the positions of free points to draw a polygon whose area approximated five square units, ultimately achieving a precise value of 4.9 square units. This dynamic adjustment process, facilitated by the dragging functionality of free geometric points within GeoGebra's AR environment, exemplifies how the tool's affordances enabled iterative refinement toward meeting the task's dual objectives of area specification and concavity. The student's strategic use of point manipulation to achieve target measurements demonstrates developed instrumented action schemes, wherein the AR tool became a genuine cognitive instrument for geometric problem-solving rather than merely a visualization device.

Afterwards, the student generated a pyramid using the "extrude pyramid" tool, selecting the base polygon and adding the height. The student initially set a height of 10 units, obtaining a volume of 16.2 cubic units. Since this value was far from the required result, the student decided to undo the action and return to the base figure. In their reasoning process, the student searched for the formula of the volume of a pyramid, which had been discussed during previous sessions, and proceeded to solve for the height to determine an approximate value. The student successfully isolated the variable and found a height of 30 units. However, to verify this "discovery", the student then constructed the pyramid with a height of 30 units, observing that the calculated volume matched the value generated by the AR tool. The student then asked the teacher the following questions: 1) "Is the height the highest side of the pyramid?", 2 "If I change the shape of the base but keep the same size, does the height stay the same?", and 3) "Is it the same with a concave polygon as with a convex one?". To answer, the teacher explained that the problem required repeating the exercise with two additional cases: a convex polygon, and a concave hexagon. Student 3 carried out both constructions with greater ease and discovered that, if the height and the area are kept constant, it does not matter whether the shape is convex or concave.

Figure 13
Cognitive scheme of Didactic Situation 2 from the workbook



4.1.5 Analysis of Didactic Situation 3

During this session, as in the previous ones, the concepts of regular polyhedral and their properties were institutionalized. With the help of AR, students identified one key property: a regular polyhedron is composed of regular polygons. The objective of this activity was to develop spatial visualization skills through the manipulation of polyhedral. The task was as follows: "Match the name with the corresponding figures and characteristics of the regular polyhedral" (see Figure 14).

In this activity, the students demonstrated complete adaptation (100%) in handling the GeoGebra 3D tool with AR. They successfully identified the geometric solids and manipulated them without

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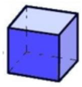
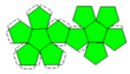
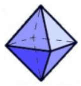
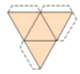
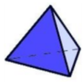
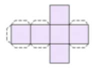
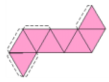
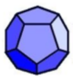
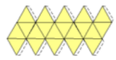
difficulty when counting their properties (vertices, edges, etc.), as well as recognizing the regular polygons that make up each polyhedron.

Figure 14
Activity corresponding to Session 3: "Regular Polyhedral"

Name	Shape	Faces	Vertices	Edges	Net
Tetrahedron		12	20	12	
Hexahedron		8	6	12	
Octahedron		20	4	30	
Dodecahedron		6	8	6	
Icosahedron		4	12	30	

4.2 Discussion

This research represents a significant contribution to the growing body of scholarship exploring the integration of emerging technologies, particularly Augmented Reality, into mathematics education. By employing GeoGebra 3D as an exploratory educational tool within a theoretically grounded pedagogical framework, this study addresses critical gaps in understanding how AR can be systematically integrated to enhance geometric reasoning and spatial skills development. The findings hold substantial implications for both academic research and practical educational applications, particularly as AR technologies become increasingly accessible and sophisticated in educational contexts.

The analysis of didactic situations throughout the intervention revealed robust evidence of active Instrumental Genesis processes, wherein students continuously modified their actions and cognitive schemes to adapt to both the affordances and constraints of the GeoGebra 3D AR tool. These dynamic adaptations confirm the theoretical propositions articulated by Artigue (2002) regarding the bidirectional nature of instrumental genesis, encompassing both instrumentalization and instrumentation processes. Students demonstrated instrumentalization by discovering and personalizing features of the AR tool, adapting visualization modes, and developing idiosyncratic strategies for manipulating virtual polyhedra. Concurrently, instrumentation was evidenced through the progressive development of systematic techniques for using AR to support geometric reasoning, including strategies for mental rotation, property identification, and spatial measurement. These findings align with recent research by Panorkou and Maloney (2016), who similarly documented instrumental genesis processes in technology-mediated geometry learning, and extend this work by demonstrating how AR specifically facilitates the development of instrumented action schemes for three-dimensional geometric reasoning.

The principal finding that it is indeed feasible to design effective didactic sequences within Mathematical Thinking, specifically in the domain of Geometry at the upper-secondary level, mediated by Augmented Reality as a dynamic geometry environment, represents a significant validation of AR's pedagogical potential. This conclusion resonates with findings from Garzón et al. (2019), whose meta-analysis of AR in educational settings demonstrated positive effects on student learning outcomes across various disciplines, with particularly strong effects in STEM domains. However, this study advances beyond generalized effectiveness claims by explicating the theoretical mechanisms through which AR supports learning, specifically through the lens of Instrumental Genesis and Didactical Engineering. The structured approach employed here, grounded in systematic pedagogical design rather than ad-hoc technology integration, addresses concerns raised by Dünser et al. (2012) regarding the need for theory-driven AR implementation in education.

The pre-test analysis illuminated critical deficiencies in students' geometric skills that are consistent with documented challenges in spatial reasoning research. Students demonstrated limited acquisition and retention of foundational geometric knowledge from previous educational levels, suggesting systemic issues in traditional geometry instruction. The most pronounced weakness identified was in mental manipulation of three-dimensional objects, particularly the ability to perform mental rotation and visualization without physical referents. This finding corroborates research by Pittalis and Christou (2010), who identified mental manipulation as a particularly challenging aspect of spatial reasoning that requires explicit instructional attention. The students' dependency on physical manipulation rather than abstract mental operations reflects what Duval (2017) describes as inadequate development of visualization and reasoning processes necessary for geometric conceptualization.

Furthermore, students' knowledge of polyhedral properties remained confined to superficial recognition without deep understanding of the structural elements constituting these geometric solids. This superficial knowledge prevented effective classification and limited students' ability to reason about relationships between different types of polyhedra. These findings align with García and López (2008), who documented how traditional metric-focused geometry instruction fails to develop robust conceptual understanding of geometric properties and relationships. The intervention's focus on interactive AR-mediated exploration specifically addressed this limitation by enabling students to manipulate, dissect, and examine polyhedra from multiple perspectives, facilitating deeper engagement with structural properties beyond mere recognition.

The observed difficulties with measurement and magnitude comprehension, particularly regarding length, area, and volume in three-dimensional contexts, reflect broader challenges in developing measurement sense documented by Battista (2007). Students' limited conceptualization of “size” in three-dimensional space suggests inadequate development of what Battista terms “structuring arrays,” the mental processes by which individuals organize and enumerate spatial units. The AR intervention addressed this challenge by providing dynamic, manipulable representations that made measurement relationships explicit and visually accessible, enabling students to develop more robust understanding of how linear, area, and volume measurements interrelate in three-dimensional objects.

Spatial orientation and positioning difficulties within three-dimensional environments represent fundamental obstacles to geometric reasoning that have been extensively documented in spatial cognition research (Newcombe & Shipley, 2015). The AR intervention's capacity to present polyhedra in augmented space, allowing students to physically move around virtual objects and view them from multiple perspectives, directly addresses these orientation challenges. This finding supports recent work by Ibáñez and Delgado-Kloos (2018), who demonstrated that AR's ability to overlay virtual objects in physical space enhances spatial understanding by providing embodied, perspectival experiences that static representations cannot offer.

The effectiveness of the AR-enhanced didactic sequence in fostering instrumental genesis and improving geometric reasoning must be understood within the broader context of technology integration research. While numerous studies have documented positive effects of technology in mathematics education, the present study's contribution lies in its systematic application of Didactical Engineering to ensure pedagogical coherence and its use of Instrumental Genesis theory to explicate cognitive processes. This theoretical grounding addresses criticisms raised by Hegedus et al. (2017) regarding atheoretical technology integration that fails to account for how learners appropriate technological tools as cognitive instruments.

The collaborative and scaffolded approach employed in this intervention, wherein students worked collectively while observing peer demonstrations, reflects socially situated perspectives on instrumental genesis. This approach acknowledges that appropriation of technological artifacts occurs not only through individual exploration but also through social interaction and the observation of expert or peer utilization schemes. This social dimension of instrumental genesis, while present in the original theoretical formulations by Rabardel (1995), has received insufficient attention in AR research and represents an important direction for future investigation.

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Looking forward, this research opens multiple avenues for continued exploration. The scalability of AR interventions, particularly regarding device compatibility issues encountered in this study, requires attention to ensure equitable access. Additionally, longitudinal investigation of whether instrumental genesis processes and geometric understanding developed through AR intervention transfer to non-AR contexts and persist over time would strengthen claims regarding AR's educational value. Finally, comparative research examining AR against other dynamic geometry environments could clarify AR's unique affordances versus general benefits of dynamic, interactive geometric visualization.

5. Conclusions

This study demonstrates that the systematic integration of Augmented Reality within carefully designed didactic sequences, grounded in the theoretical frameworks of Didactical Engineering and Instrumental Genesis, represents a viable and pedagogically sound approach to enhancing geometric reasoning and spatial skills in upper-secondary mathematics education. The research successfully addressed persistent challenges in polyhedral learning by leveraging AR's unique affordances for dynamic three-dimensional visualization and manipulation, thereby transforming abstract geometric concepts into tangible, interactive learning experiences.

The findings confirm that Instrumental Genesis processes are central to understanding how students appropriate AR technology as cognitive instruments for mathematical learning. Consistent with Artigue (2002) theoretical formulations, students demonstrated continuous modification, adaptation, and occasional abandonment of mental schemes as they engaged with increasingly complex geometric tasks. This dynamic process of scheme development was evidenced through students' flexible use of various GeoGebra tools to accomplish identical tasks, often discovering alternative construction strategies that reduced cognitive load or bypassed technical obstacles. Importantly, this instrumental flexibility emerged not solely from prior technological familiarity but rather from purposeful exploration encouraged by the pedagogical design, suggesting that well-structured didactic sequences can scaffold productive instrumental genesis even among students with limited prior experience.

The research revealed critical insights regarding the delicate balance between conceptual understanding and instrumental proficiency. Students' occasional abandonment of tasks underscores the necessity for educators and researchers to employ robust analytical methodologies capable of distinguishing between conceptual difficulties and technical manipulation challenges. This distinction is essential for designing appropriate instructional interventions and for avoiding misattribution of learning obstacles. The gradual progression of learning observed when students received consistent instrumental support validates the importance of sustained engagement with AR tools rather than isolated exposure.

Furthermore, the study highlights the crucial mediating role of teachers in facilitating instrumental genesis. Although GeoGebra provides an intuitive interface, effective utilization requires explicit instructional scaffolding regarding both the software's functionality and the underlying mathematical properties being explored. The capacity of AR-enhanced manipulation to enable students to identify and enumerate polyhedral components, faces, vertices, and edges, while connecting virtual representations to familiar physical objects demonstrates AR's potential to bridge abstract mathematical concepts with concrete, embodied experiences. This connection between virtual and physical domains represents a distinctive pedagogical affordance that traditional static representations cannot provide.

Future Work

This study opens new perspectives for the effective integration of emerging technologies in the teaching of mathematics at the upper-secondary level. It is necessary to conduct a more comprehensive study to examine how teachers adapt to the use of this tool. In the present research, the investigator also acted as the teacher, which enabled immediate resolution of difficulties to maintain the progression of the study. Based on this premise, it is proposed to carry out an Instrumental Orchestration, allowing the adaptation of schemes by teachers who are not researchers, to explore how pedagogical practices evolve when implementing Augmented Reality as a mediating tool in mathematical learning.

Limitations

This study acknowledges several limitations that should be considered when interpreting the findings. First, the reduced analytical sample size (n=12) from the initial experimental group (n=25), while methodologically justified for rigorous qualitative analysis of Instrumental Genesis processes, limits the generalizability of findings to broader populations. Second, technological constraints related to device compatibility prevented approximately 22% of students from fully participating in AR-enhanced activities, highlighting infrastructure challenges that may affect equitable implementation in diverse educational settings. Third, the study was conducted in a single upper-secondary school in Puebla, Mexico, which may limit transferability of findings to other educational contexts with different curricular structures, student populations, or technological resources. Fourth, the relatively short duration of the intervention (three sessions) provided insights into initial instrumental genesis processes but did not allow for examination of long-term retention or transfer of geometric understanding to non-AR contexts. Finally, the absence of a true control group in the quasi-experimental design limits causal claims regarding AR's specific contribution to learning outcomes compared to other potential pedagogical approaches.

Conflict of Interest
The authors declare no conflict of interest. This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The authors have no financial or personal relationships that could inappropriately influence or bias the content of this paper.

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Author Contributions
Alberto Apreza Sies: Conceptualization, methodology, investigation, formal analysis, writing—original draft preparation, and visualization.
Guillermina Sánchez-Román: Conceptualization, methodology, supervision, writing—review and editing, and validation.
José Antonio Juárez-López: Data curation, formal analysis, writing—review and editing, project administration, and manuscript submission.

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