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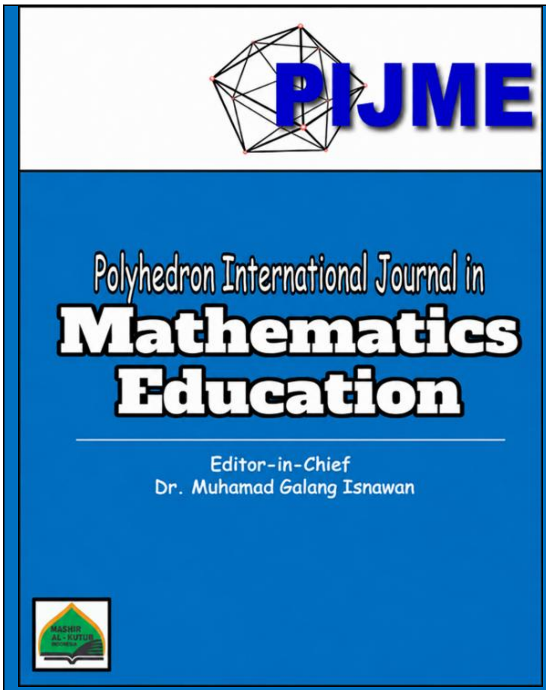

A didactical design research on think-talk-write integrated with GeoGebra: Promoting mathematical communication and collaborative learning

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Original Article

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Abstract
This study investigated the effectiveness of a Think-Talk-Write (TTW) strategy integrated with GeoGebra in promoting mathematical communication ability and collaborative learning among Grade X secondary school students. Employing a quasi-experimental design with a non-equivalent control group, the study involved 34 students in the experimental group (TTW-GeoGebra) and 32 students in the control group (conventional instruction) at SMA Negeri 3 Tanjung Raja during the 2025/2026 academic year. Data were collected using a 10-item mathematical communication test and a 15-item collaboration questionnaire, both of which demonstrated good reliability ($\alpha = 0.883$ and $\alpha = 0.866$, respectively). Results indicated that the TTW-GeoGebra model produced a moderate improvement in mathematical communication (n-gain = 0.59) and a high improvement in collaborative learning (n-gain = 0.75) in the experimental group, both significantly exceeding the control group's gains ($p < 0.001$). Between-group comparisons confirmed that the TTW-GeoGebra model was significantly more effective than conventional instruction for both outcomes. These findings suggest that embedding GeoGebra as a didactic medium across all TTW phases constitutes a principled instructional design for simultaneously developing mathematical communication and collaboration, consistent with NCTM's process standards and Indonesia's Kurikulum Merdeka.

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1. Introduction

Mathematical communication and collaboration are recognized as indispensable competencies for twenty-first-century learners. The National Council of Teachers of Mathematics (NCTM, 2000, 2014) identifies communication as one of the five process standards in mathematics education, asserting that students must be able to organize and consolidate their mathematical thinking, communicate it coherently to peers and teachers, analyze and evaluate others' mathematical reasoning, and use mathematical language with precision. Collaboration, meanwhile, is increasingly framed as both a pedagogical strategy and a learning outcome, given that cooperative problem-solving mirrors the social nature of knowledge construction and the demands of contemporary professional contexts (Greenstein, 2012). Within the Indonesian educational landscape, Kurikulum Merdeka (Kemendikbudristek, 2022) explicitly positions communication and collaboration as core twenty-first-century skills to be systematically cultivated through student-centered, active learning. Despite these curricular mandates,

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the operationalization of didactically sound learning designs that simultaneously develop both competencies remains an underexplored area, particularly at the secondary school level.

Didactical Design Research (DDR), as conceptualized by Suryadi (2010), is a design-based research approach in mathematics education that encompasses three stages: prospective analysis (identifying learning obstacles and designing instructional sequences), metapedadidactical analysis (monitoring the implementation of the design), and retrospective analysis (evaluating the alignment between the intended and actual learning trajectory). In this study, DDR serves as the overarching framework within which the TTW-GeoGebra instructional model is developed and evaluated.

Empirical evidence consistently reveals significant deficiencies in the mathematical communication abilities of Indonesian secondary school students, especially on topics involving Systems of Linear Equations and Inequalities (SPLDV). Students frequently struggle to translate contextual word problems into mathematical models, to articulate their solution procedures in oral or written form, and to represent solutions graphically (Elfin & Muthawali, 2025; Jumiati & Zanthy, 2020; Usman et al., 2021; Wati et al., 2018). These difficulties are not isolated to specific regions but have been documented across multiple provinces and school contexts throughout Indonesia (Mardayanti et al., 2016; Rahmawati et al., 2019). Parallel weaknesses in collaborative competencies have also been reported, with many students exhibiting limited participation in group discourse, low levels of shared accountability, and an inability to negotiate mathematical meaning productively (Putri et al., 2023). A preliminary investigation conducted at SMA Negeri 3 Tanjung Raja corroborated these patterns, revealing that lecture-dominated instruction without digital media support produced passive learning environments in which students' mathematical communication and collaborative engagement remained substantially underdeveloped.

Prior research has explored the potential of the Think-Talk-Write (TTW) strategy and GeoGebra as separate instructional tools to address these challenges. TTW structures student engagement through sequential phases of individual reflection, group discussion, and written documentation of reasoning, and has been demonstrated to enhance mathematical communication through this scaffolded progression (Faridah et al., 2025; Hikmah et al., 2020; Jusniani et al., 2020). GeoGebra, as a dynamic mathematics software, enables real-time visualization of symbolic and graphical representations, supporting conceptual understanding and promoting active student engagement (Afriansyah & Aini, 2025; Khaq & Wahid, 2023; Pinto et al., 2025). Some studies have examined the integration of TTW with GeoGebra and reported positive effects on conceptual understanding (Kesuma & Armanto, 2023; Rahmatika, 2022; Siregar, 2025).

The effectiveness of TTW as a communication-enhancing strategy is theoretically grounded in Vygotsky's (1978) sociocultural theory of learning, particularly the concept of the Zone of Proximal Development (ZPD). The Think phase activates each student's independent cognitive capacity; the Talk phase positions peer interaction as the mechanism through which students jointly construct mathematical meaning within their ZPD; and the Write phase consolidates this socially negotiated understanding into individual written form, reflecting the internalization process central to Vygotskian theory (Lantolf & Thorne, 2006). This theoretical alignment situates TTW not merely as a pedagogical technique but as a principled operationalization of social constructivist learning principles in the mathematics classroom.

Notwithstanding these contributions, significant gaps remain in the existing literature. Most prior studies have examined TTW and GeoGebra in isolation, or when combined, have focused exclusively on cognitive outcomes without attending to collaborative competencies. No study has simultaneously investigated the impact of a didactically designed TTW-GeoGebra integration on both mathematical communication and collaborative learning within a single intervention. The simultaneous development of communication and collaboration is theoretically warranted given their conceptual interdependence: productive collaboration presupposes mathematical communication, while mathematical communication is cultivated through meaningful collaborative interaction (Hendriana et al., 2017; NCTM, 2000).

To address these gaps, this study proposes and evaluates a didactically designed TTW-GeoGebra instructional model developed to promote mathematical communication and collaborative learning simultaneously among Grade X students on the SPLDV topic. The study is guided by three research

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questions: (RQ1) Does the TTW-GeoGebra model produce a significant improvement in students' mathematical communication ability? (RQ2) Does the TTW-GeoGebra model produce a significant improvement in students' collaborative learning ability? (RQ3) Are the improvements in both mathematical communication ability and collaborative learning significantly greater in the TTW-GeoGebra group compared to the control group?.

2. Methods

2.1 Research Design

This study employed a quasi-experimental design with a non-equivalent control group, in which participants were not randomly assigned to conditions due to existing classroom structures. This study is situated within a Didactical Design Research (DDR) framework (Suryadi, 2010), which encompasses three interconnected stages. In the prospective analysis stage, learning obstacles (didactical and epistemological) related to students' mathematical communication and collaborative engagement on the SPLDV topic were identified through a preliminary classroom investigation at SMA Negeri 3 Tanjung Raja, which revealed that conventional lecture-dominated instruction produced passive learning environments with limited communicative and collaborative development. Based on these identified obstacles, a hypothetical learning trajectory (HLT) was constructed, operationalized through the TTW-GeoGebra instructional design presented in Table 1. In the metapedadidactical analysis stage, the implementation of the design was monitored across six instructional sessions using a structured observation checklist to evaluate the alignment between the intended and enacted learning trajectory. In the retrospective analysis stage, the outcomes were evaluated through pretest-posttest comparisons and inferential statistical analyses to assess the extent to which the design achieved its intended learning objectives. This integrated DDR-quasi-experimental approach enables both the principled design and the empirical evaluation of the instructional intervention within a single study. Both groups completed pretest and posttest assessments, enabling within-group and between-group comparisons. The experimental group received the TTW-GeoGebra intervention, while the control group received conventional direct instruction on the same mathematical content (Sugiyono, 2020). The study was conducted at SMA Negeri 3 Tanjung Raja during the 2025/2026 academic year on the topic of Systems of Linear Equations and Inequalities in Two Variables (SPLDV), across six instructional sessions of 90 minutes each.

2.2 Participants

The target population comprised all 159 Grade X students enrolled in five parallel classes at SMA Negeri 3 Tanjung Raja. Purposive sampling was employed to select two classes with comparable initial mathematical ability profiles. Class selection was based on two pre-existing data sources: final examination scores from the preceding semester (Semester 1, 2024/2025 academic year) and teacher assessments of overall mathematics performance. Both classes demonstrated similar score distributions and were confirmed by the subject teacher to be at equivalent instructional levels. Formal statistical equivalence was subsequently verified using an independent samples t-test on pretest scores prior to intervention commencement ($p > 0.05$), as reported below. Class X.1 ($n = 34$; 13 male, 21 female; mean age 15 years) was assigned as the experimental group, and Class X.3 ($n = 32$; 20 male, 12 female; mean age 15 years) served as the control group. Equivalence of initial mathematical communication abilities was confirmed through independent samples t-test analysis of pretest scores ($p > 0.05$). An additional group of 28 students from outside the main sample was recruited for instrument piloting.

2.3 Intervention: The TTW-GeoGebra Didactic Design

The TTW-GeoGebra instructional model was developed as a didactically structured learning design in which GeoGebra functions as an integrated visualization and exploration medium across all three TTW phases. The operationalization of each phase is presented in Table 1.

Table 1
Operationalization of the TTW-GeoGebra Didactic Design

TTW Phase	Student Activity	GeoGebra Integration	Learning Process

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Think	Students individually read SPLDV problems and write initial mathematical ideas on structured worksheets	Students independently explore graphical representations by entering coefficients and observing real-time graphs	Individual reflection, activation of prior knowledge, mathematical modeling
Talk	Students discuss ideas in small groups (4-5 members), sharing strategies and reaching consensus on solution approaches	Groups manipulate shared GeoGebra files, adjusting parameters to visually verify the SPLDV intersection point	Collaborative discourse, peer explanation, negotiation of meaning
Write	Students document final mathematical reasoning, solution steps, and conclusions in structured written reports	Students capture GeoGebra output and integrate graphs into written explanations, connecting symbolic and visual representations	Systematic written communication, integration of representations, consolidation of understanding

The TTW-GeoGebra model was delivered by the regular subject teacher of both classes, who holds a Bachelor’s degree in Mathematics Education and has seven years of teaching experience at SMA Negeri 3 Tanjung Raja. Prior to the intervention, the teacher participated in two preparatory sessions (totalling four hours) conducted by the first author, covering: (a) the theoretical rationale and operational procedures of each TTW phase, (b) guided practice with GeoGebra for SPLDV visualization, and (c) simulation of the structured worksheet activities. Implementation fidelity was monitored using a structured observation checklist administered by an independent observer across all six instructional sessions. Observation results indicated that all core TTW-GeoGebra activities were implemented as designed: 100% of sessions fully enacted the Think and Write phases, and 100% enacted the Talk phase with GeoGebra-supported group discourse. No sessions required protocol deviation. These fidelity data confirm that the observed outcomes can be attributed to the intended instructional design rather than implementation variability.

2.4 Data Collection

Three instruments were used. The mathematical communication test comprised 10 open-ended items aligned with four indicators (Table 2). Each item was scored on a 4-point analytical rubric (0 = no response or entirely incorrect; 1 = partial understanding with major errors; 2 = adequate understanding with minor errors; 3 = complete and accurate mathematical communication), yielding a maximum possible score of 30. The rubric was adapted from the mathematical communication scoring framework developed by Hendriana et al. (2017). To establish inter-rater reliability, 20% of answer scripts (n = 13) were scored independently by two trained raters; Cohen’s Kappa coefficient was κ = 0.84, indicating strong agreement (Landis & Koch, 1977). All remaining scripts were scored by the first rater. The collaboration questionnaire consisted of 15 Likert-scale items (four-point scale) measuring five dimensions adapted from Greenstein’s (2012) framework (Table 3). A structured observation checklist monitored implementation fidelity and student engagement across instructional phases.

Table 2
Mathematical Communication Test Specifications

Indicator	Sub-Indicator	Aspect Measured
Graphical/Visual	Interpreting SPLDV graphs to identify variable relationships and solutions	Visual representation, concept interpretation, meaning-making

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	Explaining the relationship of two lines and the meaning of their intersection	Visual communication, relational understanding
Mathematical Expression	Constructing SPLDV models from contextual situations and visualizing with GeoGebra	Translation from contextual problems to symbolic and visual models
Written Text	Explaining solution procedures systematically and formulating mathematical questions	Written expression and development of mathematical arguments
Integrative	Presenting tables, graphs, and SPLDV models and explaining their interconnections	Integration of multiple mathematical communication forms

The Table 2 indicates that students' mathematical communication skills in the topic of Systems of Linear Equations in Two Variables (SPLDV) are assessed through four main indicators: graphical/visual, mathematical expression, written text, and integrative communication. The graphical/visual aspect evaluates students' ability to interpret SPLDV graphs, identify relationships between variables, and explain the meaning of the intersection point of two lines as the solution. The mathematical expression aspect measures students' competence in translating contextual situations into mathematical models and visual representations using GeoGebra. The written text aspect focuses on students' ability to explain solution procedures systematically and construct mathematical arguments. Meanwhile, the integrative aspect assesses students' ability to connect tables, graphs, and SPLDV models into a coherent and meaningful mathematical representation.

Table 3
Collaboration Questionnaire Structure

Dimension	Indicators	Items
Working productively	Active contribution, task completion	3
Respectful attitude	Appreciating peers' opinions, listening actively	3
Compromise ability	Negotiating solutions, accepting different views	3
Shared responsibility	Collective accountability for group outcomes	3
Contribution	Sharing ideas, initiating discussion, supporting peers	3
Total		15

All instruments were piloted with 28 students external to the main sample (r-table = 0.374 at $\alpha = 0.05$). Item validity was assessed using Pearson product-moment correlation; reliability was estimated using Cronbach's Alpha, with $\alpha > 0.80$ indicating good to excellent reliability (Alamsyah & Hurnaningsih, 2022). All instruments were piloted with 28 students external to the main sample. The critical value for item validity was r-table = 0.374, derived from the Pearson product-moment distribution at df = 26 (n − 2) and $\alpha = 0.05$ (two-tailed). Item validity was assessed using Pearson product-moment correlation; items with r > r-table and Sig. < 0.05 were retained. Reliability was estimated using Cronbach's Alpha; values of $\alpha \geq 0.80$ were interpreted as indicating good to excellent reliability, consistent with the threshold recommended by Nunnally and Bernstein (1994). All items were retained following piloting.

2.5 Data Analysis

Data analysis proceeded in three stages: (1) descriptive analysis of pretest and posttest score distributions; (2) normalized gain (n-gain) calculated as g = (posttest - pretest) / (maximum - pretest), classified as high ($g \geq 0.70$), moderate ($0.30 \leq g < 0.70$), or low ($g < 0.30$) (Hake, 1999); (3) inferential analysis using paired samples t-tests and independent samples t-tests, with normality (Kolmogorov-

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Smirnov) and homogeneity (Levene's test) verified at α = 0.05. All analyses were conducted using SPSS version 26. Following the recommendation of Meltzer (2002), n-gain scores were calculated for each individual student and then averaged across the group, rather than being computed from group-level mean scores. This individual-level approach avoids the potential distortion introduced when group means are used directly in the formula, as the two methods yield numerically different results that are not interchangeable.

2.7 Ethical Considerations

Formal institutional approval was obtained from the school administration of SMA Negeri 3 Tanjung Raja. Informed consent was secured from all participating students, participation was voluntary, and all data were anonymized in research outputs.

3. Results and Discussion

3.1 Results

This section presents the findings of the study in a sequential manner, beginning with the psychometric properties of the instruments, followed by assumption tests for parametric analysis, and concluding with descriptive and inferential statistical results corresponding to each research question. The presentation addresses RQ1 (within-group improvement in mathematical communication), RQ2 (within-group improvement in collaborative learning), and RQ3 (between-group comparison of gains) in systematic order.

3.1.1 Instrument Validity and Reliability

Instrument validation was conducted using data from the pilot study involving 28 students external to the main sample. Item validity was assessed using Pearson product-moment correlation, with items retained if r > r-table = 0.374 (df = 26, α = 0.05, two-tailed). All 10 items of the mathematical communication test yielded Pearson r values ranging from 0.557 to 0.846 (all Sig. < 0.05), and all 15 items of the collaboration questionnaire yielded r values ranging from 0.407 to 0.745 (all Sig. < 0.05), confirming that every item in both instruments met the validity criterion. The complete item-by-item validity results are presented in Table 4.

Table 4
Item Validity Results

Instrument	Item	Pearson r	Sig. (2-tailed)	Status
Mathematical Communication Test	TK1	0.577	0.001	Valid
	TK2	0.749	0.000	Valid
	TK3	0.773	0.000	Valid
	TK4	0.703	0.000	Valid
	TK5	0.557	0.002	Valid
	TK6	0.846	0.000	Valid
	TK7	0.644	0.000	Valid
	TK8	0.720	0.000	Valid
	TK9	0.788	0.000	Valid
	TK10	0.611	0.001	Valid
Collaboration Questionnaire	K1	0.692	0.000	Valid
	K2	0.407	0.032	Valid
	K3	0.717	0.000	Valid
	K4	0.448	0.017	Valid
	K5	0.610	0.001	Valid
	K6	0.478	0.010	Valid
	K7	0.542	0.003	Valid
	K8	0.672	0.000	Valid

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K9	0.672	0.000	Valid
K10	0.745	0.000	Valid
K11	0.627	0.000	Valid
K12	0.429	0.023	Valid
K13	0.713	0.000	Valid
K14	0.435	0.021	Valid
K15	0.672	0.000	Valid

As shown in Table 4, the Pearson r values for all items across both instruments exceeded the critical value of r-table = 0.374, with all items attaining statistical significance (Sig. < 0.05). These results confirm that every item in both instruments demonstrated sufficient discriminant validity for use in the main study.

Table 5 presents the Cronbach's Alpha reliability coefficients for both instruments, computed from the pilot data.

Table 5

Reliability Coefficients

Instrument	Cronbach's Alpha	N Items	Interpretation
Mathematical Communication Test	0.883	10	Good reliability
Collaboration Questionnaire	0.866	15	Good reliability

Both instruments attained Cronbach's Alpha values above the threshold of $\alpha \geq 0.80$ (mathematical communication test: $\alpha = 0.883$; collaboration questionnaire: $\alpha = 0.866$), confirming good reliability. The psychometric evidence from Tables 4 and 5 collectively establishes that both instruments were valid and reliable, and therefore appropriate for use as the primary measurement tools in this study.

3.1.2 Assumption Testing for Parametric Analysis

Before proceeding to hypothesis testing, the assumptions of normality and homogeneity of variance were examined to verify the suitability of parametric statistical tests. Normality was assessed using the Kolmogorov-Smirnov test, the results of which are presented in Table 6.

Table 6

Kolmogorov-Smirnov Normality Test Results

Variable	Group	K-S Stat.	df	Sig.	Conclusion
Pre-Mathematical Communication	Experimental	0.116	34	0.200	Normal
	Control	0.150	32	0.064	Normal
Post-Mathematical Communication	Experimental	0.148	34	0.056	Normal
	Control	0.146	32	0.080	Normal
Pre-Collaboration	Experimental	0.110	34	0.200	Normal
	Control	0.105	32	0.200	Normal
Post-Collaboration	Experimental	0.106	34	0.200	Normal
	Control	0.118	32	0.200	Normal

Table 6 shows that all eight variables (four variables × two groups) produced Sig. values above 0.05, with the lowest observed value being Sig. = 0.056 for the Post-Mathematical Communication score in the experimental group. This confirms that the normality assumption was met for all variables, supporting the use of parametric statistical tests.

Homogeneity of variance across groups was further examined using Levene's test, as presented in Table 7.

Table 7

Levene's Test for Homogeneity of Variance

Variable	Sig. Pretest	Sig. Posttest	Conclusion

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Mathematical Communication	0.181	0.814	Homogeneous
Collaboration	0.806	0.871	Homogeneous

Table 7 confirms that variance was homogeneous across the experimental and control groups for both mathematical communication and collaboration at both pretest and posttest (all Sig. > 0.05). With normality and homogeneity assumptions both satisfied, parametric tests — specifically paired samples t-tests and independent samples t-tests — were deemed appropriate for the subsequent hypothesis testing.

3.1.3 Descriptive Profiles - Mathematical Communication (RQ1)

To examine the nature of changes in students' mathematical communication ability beyond mean scores, the distribution of students across five achievement categories was analysed at both pretest and posttest for each group. This categorical analysis provides a more nuanced picture of learning transformation than aggregate statistics alone, as presented in Table 8.

Table 8

Distribution of Students by Mathematical Communication Category

Category	Exp. Pre-test	Exp. Posttest	Ctrl. Pretest	Ctrl. Post-test
Very High	1	11	0	2
High	0	8	0	2
Moderate	6	12	7	15
Low	5	3	9	8
Very Low	22	0	16	5
Total	34	34	32	32

As shown in Table 8, both groups were predominantly concentrated in the Very Low category at pretest, with 22 experimental students (64.7%) and 16 control students (50.0%) classified at this level, and no students in either group reaching the High or Very High categories. Following the TTW-GeoGebra intervention, the experimental group underwent a pronounced upward shift: 11 students (32.4%) reached the Very High category and 8 students (23.5%) reached High, with no students remaining in the Very Low category. In contrast, the control group showed only modest improvement, with 2 students each in Very High and High, and 5 students still classified as Very Low at posttest. Overall, the distributional data confirm that the TTW-GeoGebra intervention produced a qualitative transformation in mathematical communication performance, moving the majority of experimental students from the lowest to the upper performance levels, while the control group's distribution remained largely concentrated in the lower categories.

3.1.4 Descriptive Profiles - Collaboration (RQ2)

Similarly, to examine changes in collaborative learning, the distribution of students across four collaboration categories was analysed at pretest and posttest for both groups. Table 9 presents this distributional comparison.

Table 9

Distribution of Students by Collaboration Category

Category	Exp. Pre-test	Exp. Post-test	Ctrl. Pre-test	Ctrl. Post-test
Highly Collaborative	0	18	0	0
Collaborative	0	15	0	5
Fairly Collaborative	2	1	5	17
Not Collaborative	32	0	27	10
Total	34	34	32	32

As shown in Table 9, at pretest 32 of 34 experimental students (94.1%) were classified as Not Collaborative, with only 2 students (5.9%) in the Fairly Collaborative category. Following the intervention, a dramatic transformation occurred: 18 students (52.9%) reached the Highly Collaborative level and 15 students (44.1%) reached Collaborative, with no students remaining in the Not Collaborative category. In contrast, the control group showed substantially more limited progress, with

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5 students advancing to Collaborative and none reaching Highly Collaborative, while 10 students (31.3%) remained Not Collaborative at posttest. The near-complete elimination of the Not Collaborative category in the experimental group, alongside the absence of any Highly Collaborative students in the control group, underscores the structural advantage of the TTW-GeoGebra model in activating collaborative behaviors that conventional instruction consistently failed to stimulate.

3.1.5 Normalized Gain Analysis

To assess the magnitude of learning improvement within each group, mean normalized gain (n-gain) scores were calculated at the individual level and averaged across each group, following the recommendation of Meltzer (2002). The n-gain scores for both variables and groups are presented in Table 10.

Table 10

Mean N-Gain Scores by Variable and Group

Variable	Group	Mean N-Gain	Category
Mathematical Communication	Experimental	0.59	Moderate
	Control	0.27	Low
Collaboration	Experimental	0.75	High
	Control	0.25	Low

As shown in Table 10, the experimental group achieved a mean n-gain of g = 0.59 for mathematical communication (moderate category), compared to g = 0.27 (low category) in the control group — more than twice the control group's improvement. For collaborative learning, the difference was even more pronounced: the experimental group attained g = 0.75 (high category), compared to g = 0.25 (low category) in the control group, representing a threefold difference. These n-gain comparisons demonstrate that the TTW-GeoGebra model produced substantially greater learning gains on both variables, providing initial directional support for RQ1, RQ2, and RQ3.

3.1.6 Within-Group Hypothesis Testing - Paired t-Test (RQ1 and RQ2)

To determine whether the TTW-GeoGebra intervention produced statistically significant pre-to-post improvement within the experimental group, paired samples t-tests were conducted comparing pretest and posttest scores for both mathematical communication and collaborative learning. The results are presented in Table 11.

Table 11

Paired Samples t-Test Results (Experimental Group)

Variable	Mean Diff.	t-statistic	Sig. (2-tailed)	Conclusion
Mathematical Communication (Pre-Post)	-26.97	-10.832	0.000	Significant
Collaboration (Pre-Post)	-12.92	-12.680	0.000	Significant

As shown in Table 11, both paired t-tests yielded statistically significant results ($p < 0.001$), confirming that the TTW-GeoGebra intervention produced significant pre-to-post improvement in both mathematical communication ($t = -10.832$, mean diff. $= -26.97$) and collaborative learning ($t = -12.680$, mean diff. $= -12.92$) within the experimental group. These findings provide statistical support for RQ1 and RQ2. Having established significant within-group gains, the analysis proceeded to examine whether those gains were significantly greater than the control group's gains, as required by RQ3.

3.1.7 Between-Group Hypothesis Testing - Independent t-Test (RQ3)

To address RQ3, independent samples t-tests were conducted on the n-gain scores of both groups to determine whether the experimental group's learning gains were significantly greater than those of the control group. Table 12 presents the results of this between-group comparison.

Table 12

Independent Samples t-Test Results (Between-Group N-Gain Comparison)

Variable	t-statistic	Sig. (2-tailed)	Mean Diff.	Conclusion
Mathematical Communication	4.892	0.000	17.39	Exp > Ctrl
Collaboration	11.817	0.000	10.39	Exp > Ctrl

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Table 12 presents independent t-test results comparing n-gain scores. For mathematical communication, t = 4.892, Sig. = 0.000, mean difference = 17.39. For collaboration, t = 11.817, Sig. = 0.000, mean difference = 10.39. The experimental group's gains were significantly greater for both variables, providing strong statistical support for RQ3. Collectively, the results across all three research questions provide consistent and statistically robust evidence that the TTW-GeoGebra instructional model produced significantly greater improvements in both mathematical communication ability and collaborative learning compared to conventional instruction, with large between-group effect sizes at $p < 0.001$. These findings form the empirical basis for the interpretive discussion presented in the following section.

3.2 Discussion

The findings provide robust empirical support for the effectiveness of a didactically designed TTW-GeoGebra model in simultaneously promoting mathematical communication and collaborative learning. With respect to mathematical communication (RQ1 and RQ3), the experimental group's moderate n-gain ($g = 0.59$) and statistically significant between-group difference ($p < 0.001$, mean difference = 17.39) confirm that the TTW-GeoGebra model substantially outperformed conventional instruction. This finding is consistent with NCTM's (2000) conceptualization of communication as a process standard that must be actively cultivated through structured opportunities. The TTW structure provided this scaffold: the Think phase required individual reasoning organization; the Talk phase created discursive space for meaning negotiation; and the Write phase demanded consolidation across multiple communicative forms. GeoGebra's integration strengthened the communication process by bridging symbolic and graphical representations in real time (Afriansyah & Aini, 2025; Khaq & Wahid, 2023). These results are consistent with and extend the findings of Faridah et al. (2025), Hikmah et al. (2020), and Jusniani et al. (2020). Although the current study did not include a TTW-without-GeoGebra condition, the substantially larger gains observed in the experimental group relative to the control group suggest that the integrated TTW-GeoGebra design may produce communicative benefits that exceed what either approach could plausibly achieve in isolation. This interpretation is offered as a theoretically grounded hypothesis rather than a directly confirmed finding, and warrants empirical testing in future studies employing a three-arm design.

The collaboration results (RQ2 and RQ3) were even more pronounced ($g = 0.75$ vs. 0.25; $t = 11.817$, $p < 0.001$, mean difference = 10.39). This reflects the structural properties of TTW as a cooperative learning design. The Talk phase created conditions requiring productive peer interaction, shared reasoning, and collective accountability - the defining features of collaborative learning as theorized by Greenstein (2012). GeoGebra reinforced this collaborative dynamic by providing a shared visual object around which group discourse was organized, reducing the cognitive abstractness that can impede collaborative mathematical engagement (Askar, 2022; Pinto et al., 2025). These findings extend Nabhan et al. (2019) and Roisah et al. (2023) by demonstrating that the GeoGebra-integrated didactic design enhances collaborative outcomes beyond what TTW alone achieves.

The simultaneous development of both competencies constitutes the most theoretically significant finding, and several interpretive observations merit deeper consideration. First, the substantially larger collaboration gains ($g = 0.75$) relative to communication gains ($g = 0.59$) likely reflect differential baseline conditions: both competencies were underdeveloped at pretest, but collaborative behaviors are structurally activated more directly by the TTW format itself — the Talk phase mandates peer interaction — whereas mathematical communication improvement requires students to additionally master representational and symbolic expression, which is cognitively more demanding and slower to develop across six sessions. Second, the control group's modest but non-zero gains ($g = 0.27$ for communication, $g = 0.25$ for collaboration) suggest that conventional direct instruction retains some baseline efficacy, particularly for procedural communication tasks; however, the significantly lower effect sizes confirm that this approach is insufficient to develop the higher-order communicative and collaborative competencies prioritized by Kurikulum Merdeka (Kemendikbudristek, 2022). Third, the moderate n-gain for mathematical communication ($g = 0.59$) implies that while the TTW-GeoGebra design produced meaningful improvement, there remains substantive room for further development. Specifically, a moderate gain suggests that the six-session intervention was sufficient to initiate communicative growth but may not have been of adequate duration or intensity to move the majority of

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students to high-level communicative performance — a finding that has direct implications for instructional planning, suggesting that TTW-GeoGebra should ideally be implemented across an extended instructional unit rather than a discrete intervention block.

The simultaneous development of both competencies constitutes the most theoretically significant finding. Mathematical communication and collaboration are mutually constitutive processes: effective collaboration requires mathematical communication, while mathematical communication is enriched through collaborative interaction (Hendriana et al., 2017; NCTM, 2000). The TTW-GeoGebra model created a learning situation in which the structural interdependence of these two competencies was pedagogically instantiated, positioning it as a didactically principled design aligned with the theoretical frameworks grounding contemporary mathematics education research (Kemendikbudristek, 2022).

4. Conclusion

This study provides evidence that a TTW-GeoGebra instructional model produced statistically significant and practically meaningful improvements in both mathematical communication (n-gain = 0.59, moderate; Cohen's d ≈ 1.22 between groups) and collaborative learning (n-gain = 0.75, high; Cohen's d ≈ 2.95 between groups) relative to conventional instruction, among Grade X students at a single school site over six instructional sessions. These findings should be interpreted with appropriate caution: while the between-group differences are statistically robust ($p < 0.001$) and the effect sizes are large, the single-school quasi-experimental context limits causal inference and generalizability. The results are best understood as promising evidence warranting replication across more diverse school settings and with stronger experimental controls, rather than as conclusive proof of universal superiority.The theoretical contribution lies in demonstrating that the TTW-GeoGebra integration, when framed within a didactic mathematics perspective, constitutes a principled instructional design that operationalizes the interdependence of mathematical communication and collaboration in ways consistent with NCTM's process standards, cooperative learning theory, and Indonesia's Kurikulum Merdeka. The practical contribution of this study is threefold. First, the TTW-GeoGebra model is most directly applicable to Grade X secondary school mathematics topics involving graphical and algebraic representations — particularly Systems of Linear Equations and Inequalities (SPLDV), linear functions, and quadratic equations — where GeoGebra's dynamic visualization capability most directly supports the representational translation demands of mathematical communication. Second, successful implementation requires two prerequisite conditions: (a) student digital literacy sufficient to navigate GeoGebra's basic input functions, which can be developed through a single orientation session prior to the first instructional unit; and (b) reliable access to computers or tablets, ideally one device per group of four to five students. Teachers without prior GeoGebra experience are advised to complete a focused self-training session using freely available tutorials at geogebra.org before adopting the model. Third, the following implementation checklist summarizes the minimum conditions for effective adoption: (1) prepare structured Think-phase worksheets aligned to the lesson's SPLDV problem type; (2) configure shared GeoGebra files for each group prior to the Talk phase; (3) assign a designated scribe within each group for the Write phase; (4) allocate at least 80–90 minutes per session to allow full TTW cycle completion; and (5) monitor group discourse quality during the Talk phase using a brief observation rubric to identify groups requiring facilitative intervention.

Limitations

The single-school context limits generalizability. The quasi-experimental design does not permit causal inference with full rigor. Collaboration measurement relied exclusively on student self-report administered immediately following the intervention, introducing two potential biases: social desirability bias (students may report more positive collaboration than actually occurred) and novelty effect bias (the TTW-GeoGebra experience may have temporarily elevated students' self-assessments). Although a structured observational checklist was administered throughout the intervention to monitor implementation fidelity and student engagement, the observational data were not subjected to systematic quantitative analysis in this study; future research should incorporate observational ratings as an independent, non-self-report measure of collaboration to triangulate and validate questionnaire findings. The brief intervention period (six sessions) also raises questions about the durability of gains.Future

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research should extend the design to diverse contexts, employ stronger designs, incorporate observational methods, and examine long-term durability of gains.

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Author Contribution

Vivi Rindastri: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing - Original Draft, Editing.
Endang Wahyuningrum: Writing - Review & Editing, Supervision, Validation.
Sudirman Sudirman: Writing - Review & Editing, Formal analysis, Methodology.
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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are openly available in Zenodo at https://doi.org/10.5281/zenodo.19794213

Ethical Approval

This study was approved by the Research and Community Service Institute (LPPM) of Universitas Terbuka, Indonesia, under approval number B/1334/UN31.LPPM/PT.01.03/2026. All procedures were conducted in accordance with the Declaration of Helsinki.

Informed Consent

Written informed consent was obtained from all participants. All participants were informed of the voluntary nature of participation, their right to withdraw at any time, and data confidentiality measures.

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